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Short communication

Analog circuit optimization using adjoint network based sensitivity analysis

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ABSTRACT

Performance optimization as per the desired specifications is a major requirement of analog and mixed signal circuit design process. Rapid scaling of the semiconductor technology demands efficient optimization techniques with minimal manual efforts. In this paper, a gradient based method for analog circuit optimization using adjoint network based sensitivity analysis is presented. The sensitivity of circuit response with respect to the different parameters is computed by using analog circuit and its adjoint transformation. The proposed method is applied to optimize performance of a two stage operational amplifier (OpAmp). Subsequently, the OpAmp circuit is simulated using Cadence Virtuoso for optimized parameters and the results are validated with post fabrication measurement results.

1. Introduction

Recent advances in semiconductor technology have increased the complexity of VLSI circuits in terms of number of devices and functions. The optimization of circuits has become one of the most critical part of design process. The circuit optimization problems can be formulated as a sub-class of convex optimization problems [1]. Analyzing convex optimization problems using gradient based methods [2] and general heuristics [3] are well-known in the domain of Electronic Design Automation (EDA). Automation of electronic circuit optimization through mathematical programming is becoming increasingly difficult task with the rising level of circuit integration and complexity. The effectiveness of optimization methods [4] depends on their type and structure. These methods are prone to errors with increased number of variables, thereby limiting their performance. Increasing number of devices in VLSI systems results into a more complex mathematical formulation consisting of large number of design variables, which considerably affects the performance of optimization algorithms. Therefore, the development of alternate methods to efficiently optimize electronic circuit irrespective of its type (i.e. analog, mix-signal and RF) is an active area of research. It has been proposed that microwave circuits can be optimized using adjoint network based sensitivity analysis [5,6] which can be further employed for the optimization of analog circuits.

In this paper, an adjoint network based sensitivity analysis method, based on the Tellegen's theorem and transpose matrix computation, is proposed to evaluate gradient of the objective function with respect to design parameters of analog circuit. The steepest descent method along with Barzilai-Borwein (BB) stepsize calculation method [7] is applied to determine optimized design parameter values. Our approach simplifies the steps proposed in [5] incorporating both small signal equivalent and its adjoint transformation of analog circuit to construct sensitivity vector, which searches for any displacement in the solution space using steepest descent algorithm instead of performing any complex matrix computations (as described in [5]). The proof of concept of proposed methodology has been demonstrated in [8] for basic analog circuits. In order to illustrate the performance and feasibility of the proposed approach, a two stage op-amp circuit is optimized and fabricated using 180 nm process technology and its characterized results are compared with the simulation results to validate the proposed method.

2. Adjoint network based sensitivity analysis

To explain adjoint network based sensitivity computation, the relation between the original network and its adjoint transformation is discussed in this section. The adjoint network is derived using original electrical network and is topologically identical to it. It exhibits a reciprocal direction of signal transmission (transmittance) in comparison with the original network and has

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an incidence matrix equivalent to the transposition of incidence matrix of the original electrical network.

Let, a set of linear system of equations be $A\mathbf{x} = \mathbf{b}$. Performing absolute differentiation (neglecting higher order derivatives) on both sides, it can be written as:

$$A\delta\mathbf{x} + \delta A\mathbf{x} = \delta\mathbf{b} \;\Rightarrow\; \delta\mathbf{x} = A^{-1}[\delta\mathbf{b} - \delta A\mathbf{x}] \tag{1}$$

Assuming a scalar performance function of $\Gamma(\mathbf{x})$,

$$\delta\Gamma = \left[\frac{\partial\Gamma}{\partial\mathbf{x}}\right]^T \delta\mathbf{x} \tag{2}$$

Using (1) and (2),

$$\delta\Gamma = \left[\frac{\partial\Gamma}{\partial\mathbf{x}}\right]^T A^{-1}[\delta\mathbf{b} - \delta A\mathbf{x}] \tag{3}$$

The adjoint network can be represented by the transpose of incidence matrix representing the system of equations. Sensitivity of the response of original network with respect to parameter $\mathbf{x}$ can be expressed by formulating a vector $\boldsymbol{\eta}$ such that,

$$A^T\boldsymbol{\eta} = \left[\frac{\partial\Gamma}{\partial\mathbf{x}}\right] \;\Rightarrow\; \boldsymbol{\eta}^T = \left[\frac{\partial\Gamma}{\partial\mathbf{x}}\right]^T A^{-1} \tag{4}$$

Substituting (4) in (3),

$$\delta\Gamma = \boldsymbol{\eta}^T[\delta\mathbf{b} - \delta A\mathbf{x}] \tag{5}$$

The sensitivities of function Γ can be evaluated by solving (5). Original electrical network where the transistors are represented by their small signal equivalent circuits, is analyzed using Modified Nodal Analysis (MNA) [9]. The system matrix generated during the analysis can be described as:

$$\begin{bmatrix} A_{rG}GA_{rG}^T & A_{rT'} \\ NA_{rT'}^T & M \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_b \\ \boldsymbol{i}_{T'} \end{bmatrix} = \begin{bmatrix} -A_{rJ}\boldsymbol{i}_J \\ S_{T'} \end{bmatrix} \tag{6}$$

where $\boldsymbol{v}_b$ and $\boldsymbol{i}_{T'}$ denote vector of node potentials and branch currents in T' type branches (which represent devices other than the conductances and current sources), respectively. A_r is reduced incidence matrix of the given network ($A\mathbf{i} = 0$). Further, A_r is partitioned into three sub matrices as:

$$A_r \equiv \left(A_{rG} \vdots A_{rT} \vdots A_{rJ}\right)$$

where A_{rG}, A_{rJ}, and A_{rT} representing the conductance, current sources, and coefficients of elements other than conductances and current sources, respectively. The matrix $[M \quad N]$ represents characteristics of any device other than conductance and current sources. M, N correspond to coefficient matrices of current and voltage controlled devices, respectively. $S_{T'}$ can be expressed as:

$$[M \quad N]\begin{bmatrix} \boldsymbol{i}_{T'} \\ \boldsymbol{v}_{T'} \end{bmatrix} = S_{T'} \tag{7}$$

The formulation in (6) can be applied for transistors, transformers and all linear electrical components in the electronic circuit. The adjoint of this circuit can be represented by taking transpose of coefficient matrix as shown in (6).

$$\begin{bmatrix} (A_{rG}GA_{rG}^T)^T & -(NA_{rT'}^T)^T \\ -(A_{rT'})^T & M^T \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{v}}_b \\ \hat{\boldsymbol{i}}_{T'} \end{bmatrix} = \begin{bmatrix} \hat{I}_J \\ \hat{S}_{T'} \end{bmatrix} \tag{8}$$

where vectors $\hat{\boldsymbol{v}}_b$ and $\hat{\boldsymbol{i}}_{T'}$ represent source excitations of adjoint network. Substituting (8) in (4), we obtain the adjoint variables as,

$$\boldsymbol{\eta} = \begin{bmatrix} \hat{\boldsymbol{v}}_b \\ \hat{\boldsymbol{i}}_{T'} \end{bmatrix} \tag{9}$$

3. Proposed methodology

In this section, steps to optimize an analog circuit with respect to the design parameters using adjoint network based sensitivity approach, are presented. This approach can be applied directly to compute gradient of the objective function without any topological changes in circuit.

1. The original analog circuit is transformed into an equivalent electrical circuit by replacing MOSFETs with its small signal model, and analyzed using NgSPICE [10] which is an open source version of SPICE to evaluate DC operating point of the circuit. It should be noted that we have experimented with the initial design of circuit parameters, generated by using square law equations model of MOSFET to validate the proposed method. However, the accuracy of the solution can be improved by using higher levels of MOSFET models for small signal circuit analysis.
2. Later, adjoint of the equivalent electrical circuit obtained in step 1, is formulated. The small signal model of an NMOS transistor and its adjoint transformation are shown in Fig. 1(a) and (b), respectively. The details of this transformation is discussed in [8,11]. Adjoint transformation of original analog circuit is analyzed by NgSPICE to evaluate the DC operating point.
3. Using initial values of the design parameters, the adjoint network based sensitivity analysis is employed to evaluate the objective function in optimum direction through various iterations. Let the objective function of electrical network be $F(V_{out}, \boldsymbol{\alpha})$ where, V_{out} denotes the response of the network and $\boldsymbol{\alpha}$ denotes the vector of design parameters. In the i^{th} iteration, sensitivities of V_{out} with respect to $\boldsymbol{\alpha}$ can be expressed as:

$$\nabla F(\boldsymbol{\alpha}^i) = \frac{\partial F}{\partial\boldsymbol{\alpha}^i} = \beta\left[\frac{\partial V_{out}^i}{\partial\boldsymbol{\alpha}^i}\right] = \beta\left[\frac{\partial\Gamma^i}{\partial\boldsymbol{\alpha}^i}\right] \tag{10}$$

where $\frac{\partial V_{out}^i}{\partial\boldsymbol{\alpha}^i}$ is equivalent to the expression given in (3) and β is a functional relation between the circuit response and objective function. From Eq. (6), (8), and (10), sensitivity of the output voltage [12] can be expressed as:

$$\left[\frac{\partial V_{out}^i}{\partial\boldsymbol{\alpha}^i}\right] = \begin{bmatrix} \boldsymbol{v}_b \\ \boldsymbol{i}_{T'} \end{bmatrix}^T \begin{bmatrix} \frac{\partial(A_{rG}GA_{rG}^T)^T}{\partial\boldsymbol{\alpha}^i} & -\frac{\partial(NA_{rT'}^T)^T}{\partial\boldsymbol{\alpha}^i} \\ -\frac{\partial(A_{rT'})^T}{\partial\boldsymbol{\alpha}^i} & \frac{\partial M^T}{\partial\boldsymbol{\alpha}^i} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{v}}_b \\ \hat{\boldsymbol{i}}_{T'} \end{bmatrix} \tag{11}$$

For $\boldsymbol{\alpha} = \left[A_{rG}GA_{rG}^T\right]^T$ (perturbation across conductance elements), the gradient $\frac{\partial V_{out}^i}{\partial\boldsymbol{\alpha}^i}$ can be formulated as:

$$\left[\frac{\partial V_{out}^i}{\partial\boldsymbol{\alpha}^i}\right] = [\,\boldsymbol{v}_b\hat{\boldsymbol{v}}_b\,]^T \tag{12}$$

Thus, the sensitivities of output response can be calculated by the solution (node voltages and branch currents) of original and adjoint circuits.

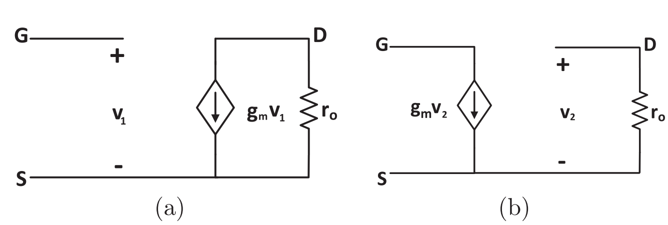

Fig. 1. Small signal equivalent of an (a) NMOS transistor and (b) its adjoint transformation.