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A covariance matrix shrinkage method with Toeplitz rectified target for DOA estimation under the uniform linear array

Yanyan Liu [a], Xiaoying Sun [a,*], Shishun Zhao [b]

[a] Department of Communication Engineering of Jilin University, Changchun, Jilin 130025, China
[b] Department of Mathematics of Jilin University, Changchun, Jilin 130022, China

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A B S T R A C T

A covariance matrix shrinkage method is proposed to make an improvement of the direction of arrival (DOA) estimation under a uniform linear array in a scenario where the number of sensors is large and the sample size is relatively small. The main contribution is that we provide a shrinkage target with Toeplitz structure and deduce a closed-form estimation of the shrinkage coefficient. The closed-form and the expectation of the shrinkage coefficient estimate are calculated based on the unbiased and consistent estimates of the trace and moments of a Wishart distributed covariance matrix. The statistical property of the shrinkage coefficient estimate is discussed through theoretical analysis and simulations, which demonstrate the shrinkage coefficient estimate can ensure that the proposed covariance matrix estimate is a good compromise between the sample covariance matrix (SCM) and the target. The root-mean-square-error (RMSE) simulations of DOA estimation show that the proposed method can improve the multiple signal classification (MUSIC) DOA estimation performance in the case of low signal-to-noise ratio (SNR) with small sample size, and also can provide a satisfactory performance at high SNR.

1. Introduction

Direction-of-arrival (DOA) estimation of signal sources is a fundamental topic in signal processing and widely applied in communications, radar and sonar, etc. [1,2]. The subspace-based DOA estimation methods including multiple signal classification (MUSIC) algorithm and the modified versions are referred to as super-resolution techniques in the case of high signal-to-noise ratio (SNR) with sufficient samples, which offer a good compromise between resolution and computational complexity [3,4]. They obtain the signal and noise subspaces through the eigenvalue decomposition of the sample covariance matrix (SCM). The SCM in these methods is usually estimated by the maximum likelihood estimation, which is a well estimation when the sample size is much lager than the dimension. However, in certain scenarios, the number of available samples N may be restricted and is on the same order of magnitude as the number of sensors M. For example, when signals are short-time stationary processes, or an array system contains a large number of sensors in the multiple-input multiple-out (MIMO) radar system. In these cases, the SCM is not a good estimation of the true covariance matrix any more, which leads to the subspace-based DOA estimation methods perform poorly [5].

The general asymptotic situation, where $M, N \to \infty$ with $M/N \to c \in (0, \infty)$, can provide a more accurate description for the practical scenario in which M and N are the finite with comparable values [6]. X. Mestre analyzed the asymptotic behavior of the eigenvalues and eigenvectors of the SCM by Stieltjes transform, and proved that the traditional sample estimates are inconsistent and indicate a poor performance in the general asymptotic situation [7]. His team modified the subspace algorithms for DOA estimation (their methods are named as G-MUSIC and G-SSMUSIC) based on their improved estimation of the eigenvalues and eigenvectors [8]. Compared with the conventional subspace methods, X. Mestre et al. focused on providing new estimations of the quadratic form of the eigenvectors and improving the resolution of DOA estimation by weighting the sample eigenvector projection matrices. From another perspective, it will be an effective way to obtain good DOA estimates via improving the estimation of the covariance matrix when N is relatively small compared with M.

Covariance matrix shrinkage estimation algorithms are suitable for high dimensional problems with relatively few samples (large M and small N), and the estimate realizes a good compromise between the SCM and a well-conditioned matrix (the

* Corresponding author.
E-mail addresses: liuyyjany@126.com (Y. Liu), sunxy@jlu.edu.cn (X. Sun), zhaoss@jlu.edu.cn (S. Zhao).

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shrinkage target) [9,10]. The researches about shrinkage methods focused on finding a proper shrinkage target, and a shrinkage coefficient which should be optimal and easy to calculate. Recently, Y. Chen et al. addressed a high dimensional covariance matrix shrinkage method in the sense of minimum mean-squared-error (MSE) when the observations are Gaussian distributed [11] and elliptical distributed [12]. X. chen and Z. J. Wang introduced a shrinkage-to-tapering approach which shrinks the SCM to the tapered version by choosing some diagonals of the SCM [13]. T. Lancewicki and M. Aladjem considered a multi-target shrinkage algorithm which exploits the Ledoit-Wolf (LW) method with several targets simultaneously [14]. Most of these methods assume that the true covariance matrix likes an identity matrix, a diagonal matrix, or a diagonally dominant matrix with smoothing parameters [13–15]. Under a general assumption of the DOA estimation model, the true covariance matrix is a complex Toeplitz and Hermitian matrix with coherent entries under the uniform linear array (ULA) or other fixed structure matrices depending on the geometry of the array. Among the above mentioned methods, the diagonal matrix structure targets adopt the entries in the main diagonal of the SCM and do not contain any information about the DOAs, and the others drop some DOA information from the minor diagonals of the SCM. Hence, the current covariance matrix shrinkage methods are not suitable for the DOA estimation model. Because the shrinkage targets of these methods are not well-conditional matrices compared with the covariance matrix of the DOA estimation model.

The Toeplitz rectification is a way to improve the estimation of a covariance matrix with Toeplitz structure, which obtains a rectified SCM by averaging the entries on the diagonals of the SCM [16,17]. The R-MUSIC improved the DOA estimation of MUSIC by replacing the SCM with the Toeplitz rectified SCM. P. Vallet and P. Loubaton proved that the R-MUSIC suffers a "saturation phenomenon" that the MSE of DOA estimates will not decrease with the increase of signal-to-noise ratio (SNR) when the SNR overs a certain value [17]. Although the rectified SCM with the flaw of "saturation phenomenon", it is suitable as a shrinkage target due to its Toeplitz structure. In this paper, we consider the rectified SCM as the shrinkage target with the advantage that it contains all DOA information and provides a good DOA estimation performance at low SNR. Utilizing the unbiased and consistent estimates of the trace and moments of the Wishart distributed covariance matrix, the estimation of the shrinkage coefficient is derived as a closed form. The proposed shrinkage coefficient is inversely proportional to the SNR and tends to a stable value with the increase of the number of samples, which means when the SNR is high, the SCM accounts for a major share in the new covariance matrix estimate and the "saturation phenomenon" from the rectified SCM will be mitigated. On the contrary, the rectified SCM will play a leading role and bring a good DOA estimation performance when the SNR is low and the sample size is small.

The rest of the paper is organized as follows. The signal model, the MUSIC and G-MUSIC algorithms are presented in Section 2. The proposed covariance matrix shrinkage estimation method, the application in the MUSIC and the statistical analysis of the shrinkage coefficient estimate are introduced in Section 3. Numerical simulation results are shown in Section 4. The principal conclusion is summarized in Section 5.

Notation. In the following, we depict vectors in lowercase boldface letters and matrices in uppercase boldface. The transpose operator and conjugate transpose operator are denoted as $(\cdot)^T$ and $(\cdot)^H$, respectively. $Tr(\cdot)$, $E\{\cdot\}$ and $\|\cdot\|_F$ are the trace, the mathematical expectation and the Frobenius norm, respectively.

2. The signal model, MUSIC and G-MUSIC algorithms

2.1. The signal model

In consideration of a ULA of M sensors with half-wavelength element separation receiving K narrow-band spatially incoherent signals from directions $\{\theta_1,\dots,\theta_K\}$, at discrete time n, the received sample vector $\boldsymbol{y}(n)\in\mathbb{C}^{M\times 1}$ is usually modeled as

$$\begin{aligned}\boldsymbol{y}(n) &= \sum_{k=1}^{K}\boldsymbol{a}(\theta_k)s_k(n)+\boldsymbol{w}(n)\\ &= \boldsymbol{As}(n)+\boldsymbol{w}(n),\end{aligned}\tag{1}$$

where $\boldsymbol{A}=[\boldsymbol{a}(\theta_1),\dots,\boldsymbol{a}(\theta_K)]\in\mathbb{C}^{M\times K}$ is the steering matrix with unit norm steering vectors $\boldsymbol{a}(\theta_k)=\frac{1}{\sqrt{M}}[1,e^{j\pi\sin\theta_k},\dots,e^{j\pi(M-1)\sin\theta_k}]^T$, $k=1,\dots,K$, and $\boldsymbol{s}(n)=[s_1(n),\dots,s_K(n)]^T\in\mathbb{C}^{K\times 1}$ contains source signals, and $\boldsymbol{w}(n)\in\mathbb{C}^{M\times 1}$ is the additive noise. We assume there are N samples collected in the sample matrix

$$\boldsymbol{Y}_N=\boldsymbol{AS}_N+\boldsymbol{W}_N,\tag{2}$$

where $\boldsymbol{Y}_N=[\boldsymbol{y}(1),\dots\boldsymbol{y}(N)],\boldsymbol{S}_N=[\boldsymbol{s}(1),\dots\boldsymbol{s}(N)]$, and $\boldsymbol{W}_N=[\boldsymbol{w}(1),\dots\boldsymbol{w}(N)]$. We consider common assumptions of the model as following.

A 1. The signals are independent with mean $E\{s_k(n)\}=0$, $k=1,\dots,K$ and covariance matrix $E\{\boldsymbol{s}(n)\boldsymbol{s}^H(n)\}\triangleq\boldsymbol{P}_s$.

A 2. $\boldsymbol{w}(n)$ is the complex white Gaussian noise with zero mean and unknown power σ^2, i.e. $E\{\boldsymbol{w}(n)\boldsymbol{w}^H(n)\}=\sigma^2\boldsymbol{I}_M$, where $\boldsymbol{I}_M$ is an $M\times M$ identity matrix. The noise is independent of the signals.

A 3. The number of sources K is known and satisfies $K<min(M,N)$.

Under the assumptions, the true covariance matrix of the observation vector $\boldsymbol{y}(n)$ is

$$\boldsymbol{R}=E\{\boldsymbol{y}(n)\boldsymbol{y}^H(n)\}=\boldsymbol{AP}_s\boldsymbol{A}^H+\sigma^2\boldsymbol{I}_M.\tag{3}$$

We denote the eigenvalues of $\boldsymbol{R}$ as $\lambda_1\leqslant\lambda_2\leqslant\cdots\leqslant\lambda_M$ with the corresponding eigenvectors $\boldsymbol{e}_1,\dots,\boldsymbol{e}_M$. The sample covariance matrix (SCM) $\boldsymbol{S}=\frac{1}{N}\boldsymbol{Y}_N\boldsymbol{Y}_N^H$is the classical maximum likelihood estimation of $\boldsymbol{R}$, but in this paper we consider an unbiased estimate

$$\hat{\boldsymbol{S}}=\frac{1}{N-1}\boldsymbol{Y}_N\boldsymbol{Y}_N^H.\tag{4}$$

The eigenvalues and eigenvectors of $\hat{\boldsymbol{S}}$ are denoted as $\hat{\lambda}_1\leqslant\hat{\lambda}_2\leqslant\cdots\leqslant\hat{\lambda}_M$ and $\hat{\boldsymbol{e}}_1,\dots,\hat{\boldsymbol{e}}_M$, respectively, also called as sample eigenvalues and sample eigenvectors. Consequently, the DOA estimation is to infer the parameters $\theta_k,k=1,\dots,K$ from the noisy observation matrix $\boldsymbol{Y}_N$.

2.2. The MUSIC and G-MUSIC algorithms

Under the Assumptions A1–A3, the MUSIC algorithm is based on the fact that $\theta_1,\dots,\theta_K$ are the zeros of the pseudo-spectrum function

$$\eta_{MUSIC}(\theta)=\boldsymbol{a}^H(\theta)\boldsymbol{\Pi}\boldsymbol{a}(\theta),\tag{5}$$

where $\boldsymbol{\Pi}=\sum_{m=1}^{M-K}\boldsymbol{e}_m\boldsymbol{e}_m^H$ is the orthogonal projection matrix onto the kernel of $\boldsymbol{AP}_s\boldsymbol{A}^H$, and also called as "noise subspace projection matrix" [3]. The unknown matrix $\boldsymbol{\Pi}$ is usually obtained by computing the eigenvectors associated with the $M-K$ smallest eigenvalues of the SCM, i.e. $\hat{\boldsymbol{\Pi}}_{SCM}=\sum_{m=1}^{M-K}\hat{\boldsymbol{e}}_m\hat{\boldsymbol{e}}_m^H$. The original MUSIC estimates