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Exact analytical inverse mapping of decomposable TS fuzzy systems with singleton and linear consequents

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ABSTRACT

In this study, a new method is proposed for the exact analytical inverse mapping of Takagi–Sugeno fuzzy systems with singleton and linear consequents where the input variables are described by using strong triangular partitions. These fuzzy systems can be decomposed into several fuzzy subsystems. The output of the fuzzy subsystem results in multi-linear form in singleton consequent case or multi-variate second order polynomial form in linear consequent case. Since there exist explicit analytical formulas for the solutions of first and second order equations, the exact analytical inverse solutions can be obtained for decomposable Takagi–Sugeno fuzzy systems with singleton and linear consequents. In the proposed method, the output of the fuzzy subsystem is represented by using the matrix multiplication form. The parametric inverse definition of the fuzzy subsystem is obtained by using appropriate matrix partitioning with respect to the inversion variable. The inverse mapping of each fuzzy subsystem can then easily be calculated by substituting appropriate parameters of the fuzzy subsystem into this parametric inverse definition. So, it becomes very easy to find the analytical inverse mapping of the overall Takagi–Sugeno fuzzy system by composing inverse mappings of all fuzzy subsystems. The exactness and the effectiveness of the proposed inversion method are demonstrated on trajectory tracking problems by simulations.

1. Introduction

The inversion of a system model has an important role in model based engineering applications. However, in general, it is not easy to obtain a mathematical model for nonlinear systems. Since fuzzy systems are universal approximators which can approximate any nonlinear function with an arbitrary degree of accuracy [1–5], they are easy and effective tools for modeling nonlinear systems. Therefore, the fuzzy model inversion is widely used especially in model based engineering applications [6–14].

In literature, there are several methods which provide exact or approximate inverse solutions for fuzzy system models. The exactness of the solution is important in order to ensure stability or robustness of the process. However, the exact inversion methods need certain limitations on the fuzzy system to be inverted such as having monotonic rule bases, singleton consequents, invertibility property. On the other hand there are no such limitations for the approximate inversion methods.

The inversion of a fuzzy system can be handled as an identification problem manner and the approximate inverse fuzzy model can be obtained directly by using the reverse input–output data of the system [15–17]. In a similar manner, the inversion of a system can also be handled as an optimization problem manner and the appropriate inversion variable minimizing error between the desired and the system output can be found by using iterative procedures such as Newton method [18], Levenberg–Marquardt algorithm [19], genetic algorithms [20], Big Bang-Big Crunch algorithm [21]. In these iterative methods, it is difficult to guarantee the desired convergence in each sampling time for any nonlinear system. Therefore, sufficient convergence may not be obtained in each sampling time and this reduces the stability and the robustness of the process for practical implementations.

On the other hand, the exact inverse fuzzy model of a fuzzy system can be obtained by using rule-by-rule inversion approach which permutes the antecedent and consequence parts of the fuzzy model [9,22–25]. This linguistic inversion approach is valid for the fuzzy models of which input variables are defined by using strong triangular fuzzy partitioning and rule consequents are singletons. Additionally, some

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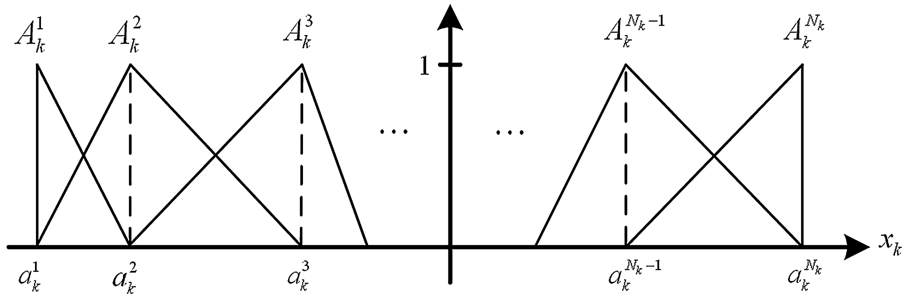

Fig. 1. Strong triangular fuzzy partition.

invertibility conditions which involve the monotonicity of the entire rule base of the fuzzy model must be satisfied. These invertibility conditions guarantee the existence and uniqueness of the inverse solution. In [26], the exact inversion method which handles the inversion procedure partially is proposed for decomposable fuzzy systems. Takagi–Sugeno (TS) fuzzy systems of which input variables are defined by using strong triangular partition and the rule consequents are singletons can be decomposed into multi-linear subsystems [27]. In this method, the inverse solution for overall fuzzy system is obtained by calculating inverse solutions of its all subsystems individually. Therefore, there is no need to check the invertibility of the fuzzy system beforehand. However, this approach is valid only for decomposable fuzzy systems with singleton consequents. In [28], an exact inversion method which removes the necessity of the decomposability property is proposed for TS fuzzy systems with singleton consequents. The only limitation of this method is that the inversion variable must be represented by using piecewise linear membership functions. In [29], an analytical method is proposed for the exact inversion of TS fuzzy systems with linear consequents where the input variables are described using strong triangular partitions. In [30], the inversion method proposed in [28] is extended to the inversion of fuzzy systems with linear consequents.

In this study, a new exact analytical inverse mapping method is proposed for TS fuzzy systems with singleton and linear consequents where the input variables are described by using strong triangular partitions. TS fuzzy systems having strong triangular partitions can be decomposed into several fuzzy subsystems. In the proposed method, the analytical formulation of the output of each fuzzy subsystem is derived by using the matrix multiplication form representation of decomposable TS fuzzy systems. This matrix multiplication form simplifies the mathematical representation of the overall TS fuzzy system and also its inversion procedure especially in the case of high number of input variables. The output of the fuzzy subsystem results in multi-linear form in singleton consequent case or multi-variate second order polynomial form in linear consequent case. Thus, first and second order equations are derived to be solved by using appropriate matrix partitioning for the fuzzy systems with singleton and linear consequents, respectively. Then, the parametric inverse definition of the fuzzy subsystem is obtained via analytical formulas. The inverse mapping of each fuzzy subsystem can then easily be calculated by substituting appropriate parameters of the fuzzy subsystem into this parametric inverse definition. So, it becomes very easy to find the analytical inverse mapping of the overall TS fuzzy system by composing inverse mappings of all fuzzy subsystems. Algorithms are also given for the practical implementation of the proposed inversion procedures. The exactness and effectiveness of the proposed inversion method are demonstrated on trajectory tracking problems by simulations.

The rest of the paper is organized as follows: In *Decomposition of TS fuzzy systems* section, decomposition of TS fuzzy systems is briefly presented. In *The exact inversion of decomposable TS fuzzy systems* section, the proposed exact inversion method is introduced. In *Simulation Studies* section, simulation studies are given to show the exactness and the effectiveness of the proposed method. Finally, conclusions are outlined in Conclusion section.

2. Decomposition of TS fuzzy systems

The general rule structure of a TS fuzzy system with n input variables, $x_k \in X_k \subset \Re$, $k = 1, ..., n$, and one output variable, $y \in Y \subset \Re$, can be defined as [26]

$$\begin{aligned} & R^{i_1,i_2,\ldots,i_n} : \text{IF } x_1 \text{ is } A_1^{i_1} \text{ and } x_2 \text{ is } A_2^{i_2} \text{ and}, \ldots, \text{and } x_n \text{ is } A_n^{i_n} \\ & \text{THEN } y = f^{i_1,i_2,\ldots,i_n}(x_1, x_2, \ldots, x_n) \end{aligned} \quad (1)$$

where $A_k^{i_k}$, $k = 1, \ldots, n$, is the fuzzy set defined for the input variable x_k and $f^{i_1,i_2,\ldots,i_n}$ is the consequent crisp function. When N_k fuzzy sets are used for the definition of x_k, $\{i_k \in I_k = \{1, 2, \ldots, N_k\}\}$, $k = 1, \ldots, n$, the complete rule base consists of $N = \prod_{k=1,\ldots,n} N_k$ rules with the corresponding index set $I = I_1 \times I_2 \times, \ldots, \times I_n$. In singleton and linear consequent cases, the crisp rule output functions are defined as $f^{i_1,i_2,\ldots,i_n} = q^{i_1,i_2,\ldots,i_n}$ and $f^{i_1,i_2,\ldots,i_n} = q_0^{i_1,i_2,\ldots,i_n} + q_1^{i_1,i_2,\ldots,i_n} \cdot x_1 + \cdots + q_n^{i_1,i_2,\ldots,i_n} \cdot x_n$, respectively. Here, $q_j^{i_1,i_2,\ldots,i_n}$ $(j = 0, \ldots, n)$, are the coefficients of output crisp functions.

Strong triangular fuzzy partition is commonly used in fuzzy system design for the definition of input variables. Because, such fuzzy partitions are easy to construct, have a clear interpretation and, most importantly, are sufficient for modeling complex highly nonlinear systems [31].

When strong triangular fuzzy partition shown in Fig. 1 is used for the definition of input variables, each universe of discourse $X_k = [a_k^1, a_k^{N_k}]$, $k = 1, \ldots, n$ can be considered as the union of $N_k - 1$ intervals defined by two consecutive model values as

$$X_k = \bigcup_{i_k=1,\ldots,N_k-1} \left[a_k^{i_k}, a_k^{i_k+1}\right] \quad k = 1, \ldots, n \quad (2)$$