Int. J. Electron. Commun. (AEÜ) 76 (2017) 77–85

Contents lists available at ScienceDirect

International Journal of Electronics and Communications (AEÜ)

journal homepage: www.elsevier.com/locate/aeue

Regular paper

Widely linear generalized sidelobe canceling beamforming with variable diagonal loading

Aimin Song [a,*], Aiqi Wang [a], Shengyang Luan [b], Tianshuang Qiu [b]

[a] School of Science, Dalian Jiaotong University, Dalian 116028, China
[b] Faculty of Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian 116024, China

ARTICLE INFO

Article history:
Received 5 October 2016
Accepted 31 March 2017

Keywords:
Adaptive beamforming
Widely linear
Mismatch
Variable diagonal loading

ABSTRACT

We propose a robust widely linear (WL) beamformer for noncircular (NC) signals in the presence of angle of arrival (AOA) errors or array random perturbations. In our beamformer, the block conjugate structure of covariance matrix is exploited to avoid updating the full weight vector, which reduces the computational loads. We add a variable diagonal loading term in the weight vector to improve the robustness of the WL beamformer. Moreover, the orthogonality constraint of block matrix is not required to calculate the amount of diagonal loading. Computer-simulation results show that the proposed WL beamforming provides improved performance over the conventional linear beamformers for NC signals in the presence of AOA errors and array random perturbations.

1. Introduction

Beamforming is a versatile approach to enhance the desired signal and alleviate the interference and noise in array processing. It has played an important role in many applications such as radar, sonar, and wireless communications [1–3]. The linearly constrained minimum power (LCMP) beamforming has been widely used in beamforming design [1]. It selects the weight vector adaptively to minimize the array output power subject to the linear constraint. Therefore LCMP beamforming has good resolution and interference rejection capability [1,4].

Generalized sidelobe canceling (GSC) beamforming is an important approach to implement the LCMP beamforming [1,4–6]. It can effectively reduce the computational costs, especially implemented with adaptive algorithms [5,6]. Furthermore, it uses an unconstrained rather than a constrained algorithm to adapt the weights, which converges faster [1,5,6].

Numerous LCMP beamformer algorithms have been proposed in the framework of GSC structure. For example, a robust GSC beamformer with feedback filter was proposed to deal with the mismatch between the estimated and actual angle of arrival (AOA) in [5]. However, when the directions of the desired signal and interference sources change over time, direct application of this GSC to the non-stationary signal environment may lead to divergence. Lee et al. [7] combined the Householder transformation and least mean square algorithm for the feedback GSC, which was effective in non-stationary scenario. In the framework of GSC structure, Tian et al. [8] presented LCMP beamforming with a quadratic inequality constraint against the angle of arrival (AOA) errors and array random perturbations. In their approach, a variable diagonal loading term was added to improve the robustness of the beamformer. However, they considered linear beamformer, which is not optimal for the noncircular (NC) signals. And the blocking matrix is assumed to be orthogonal for calculating the diagonal loading term. These motivate our work.

We propose a widely linear (WL) variable diagonal loading beamforming to improve the performance for the mismatch of noncircular (NC) signals. NC signals, such as binary-phase-shift-keying(BPSK), amplitude-shift keying(ASK), minimum-shift-keying(MSK) and Gaussian-minimum-shift-keying (GMSK), have been widely used in modern communication systems [9,10,12]. In order to improve the performance of beamforming for the NC signals, we adopt the WL technique, i.e., both the original signal and its conjugate analogue are considered to propose the beamformers [9–11,13,14]. Moreover, no orthogonality constraint is required for the block matrix in our method and the computational costs of our beamformer are reduced by avoiding updating the full matrix. Therefore, our method can be regarded as the generalization of [8] to NC signals.

The remainder of this paper is organized as follows. In Section 2, we present the LCMP beamforming and its GSC implementation. In Section 3, we present the computationally-efficient WL recursive

* Corresponding author.
E-mail addresses: songaimin@djtu.edu.cn (A. Song), aiqiwang@126.com (A. Wang), luanshengyang@live.com (S. Luan), qiutsh@dlut.edu.cn (T. Qiu).

http://dx.doi.org/10.1016/j.aeue.2017.03.025
1434-8411/

least squares beamformer in GSC structure. Based on this computationally-efficient beamformer, we propose the variable diagonal loading WL beamformer against the mismatch for the NC signals in Section 4. At the end of this section, we compare the proposed approach with other beamformers in term of computational complexity. The performance and parameter discussion are provided in Section 5. Conclusions are drawn in Section 6.

The following notations are used throughout the paper. Matrices and vectors are represented by bold upper-case and bold lower-case characters, respectively. The superscripts $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^{-1}$ stand for conjugate, transpose, conjugate transpose and matrix inverse, respectively. $\boldsymbol{I}_M$ and $\boldsymbol{0}_{M\times L}$ denote the $M\times M$ identity matrix and $M\times L$ zero matrix. The $\|\cdot\|$ stands for the Euclidean norm. The $\Re[\cdot]$ denotes the real part of a complex.

2. GSC beamforming

Consider a uniform linear array with M omnidirectional sensors and an adjacent sensor spacing d. Suppose that there are $J+1(J+1\leqslant M)$ narrowband signals with wavelength τ impinging upon the arrays from the far field with angles $\theta_0,\theta_1,\ldots,\theta_J$. Then the kth snapshot vector $\boldsymbol{x}(k)$ can be described by [1,8]

$$\boldsymbol{x}(k)=s_0(k)\boldsymbol{a}(\theta_0)+\sum_{j=1}^{J}s_j(k)\boldsymbol{a}(\theta_j)+\boldsymbol{v}(k),\quad k=1,2,\ldots,N \tag{2.1}$$

where $\boldsymbol{x}(k)=[x_1(k),x_2(k),\ldots,x_M(k)]^T$ is the sensor output vector. Here $s_0(k)$ and $s_j(k), j=1,2,\ldots,J$ are the signal of interest (SOI) and the jth interference signals at instant k,respectively. The vector $\boldsymbol{a}(\theta_j)=[1,e^{i\Psi_j},\ldots,e^{i(M-1)\Psi_j}]^T$ is the array steering vector where $\Psi_j=\frac{2\pi}{\tau}d\sin(\theta_j), j=0,1,\ldots,J$. The vector $\boldsymbol{v}(k)$ is the noise vector denoted by $\boldsymbol{v}(k)=[v_1(k),v_2(k),\ldots,v_M(k)]^T$.

LCMP has been widely used in beamforming design. It can be formulated as the following optimization problem [1,8]

$$\boldsymbol{w}=\arg\min\ \boldsymbol{w}^H\boldsymbol{R}_x\boldsymbol{w}\quad \text{s.t.}\quad \boldsymbol{C}^H\boldsymbol{w}=\boldsymbol{f}, \tag{2.2}$$

where $\boldsymbol{R_x}=E[\boldsymbol{x}(k)\boldsymbol{x}^H(k)]$ is the covariance matrix, $\boldsymbol{C}$ is a $L\times M$-dimensional constraint matrix and $\boldsymbol{f}$ is a $L\times 1$-dimensional vector. GSC beamforming is an important approach to implement the LCMP beamforming. In the GSC beamforming, the weight $\boldsymbol{w}$ is written as the combinations of $\boldsymbol{w}_q$ and $\boldsymbol{w}_a$ by

$$\boldsymbol{w}=\boldsymbol{w}_q-\boldsymbol{B}\boldsymbol{w}_a, \tag{2.3}$$

where the fixed weight

$$\boldsymbol{w}_q=\boldsymbol{C}\left(\boldsymbol{C}^H\boldsymbol{C}\right)^{-1}\boldsymbol{f} \tag{2.4}$$

is the component in the constraint subspace, and the adaptive weight $\boldsymbol{w}_a$ is updated by the optimal problem

$$\min\ E\left[|y_p(k)|^2\right], \tag{2.5}$$

where

$$y_p(k)=\boldsymbol{w}_q^H\boldsymbol{x}(k)-\boldsymbol{w}_a^H\boldsymbol{z}(k)=\left(\boldsymbol{w}_q-\boldsymbol{B}\boldsymbol{w}_a\right)^H\boldsymbol{x}(k). \tag{2.6}$$

The full column rank $M\times(M-L)$-dimensional matrix $\boldsymbol{B}$ in (2.3) is the blocking matrix which is orthogonal to the constraint matrix $\boldsymbol{C}$, i.e., $\boldsymbol{B}^H\boldsymbol{C}=\boldsymbol{0}_{(M-L)\times L}$. If the blocking matrix $\boldsymbol{B}$ satisfies the orthogonal condition $\boldsymbol{B}^H\boldsymbol{B}=\boldsymbol{I}_{(M-L)}$, the matrix $\boldsymbol{B}$ is called the orthogonal blocking matrix.

3. Widely linear GSC beamforming

Although the conventional adaptive beamformer has been shown to be optimal for circular sources [1], it becomes suboptimal for NC signals because the properties of the NC signals have not yet been completely exploited [10,12]. As a remedy, both the covariance $\boldsymbol{R_x}=E[\boldsymbol{x}(k)\boldsymbol{x}^H(k)]$ and the complementary covariance matrix $\boldsymbol{Q_x}=E[\boldsymbol{x}(k)\boldsymbol{x}^T(k)]$ are taken into account in WL beamforming. To exploit the information of $\boldsymbol{Q_x}$, we define the augmented vector $\tilde{\boldsymbol{x}}(k)$ by combing $\boldsymbol{x}(k)$ and its complex conjugate $\boldsymbol{x}^*(k)$ as follows:

$$\tilde{\boldsymbol{x}}(k)=\left(\boldsymbol{x}^T(k),\boldsymbol{x}^H(k)\right)^T \tag{3.1}$$

Then the output of the branch via the weight $\tilde{\boldsymbol{w}}_q$ can be written as

$$\tilde{y}_c(k)=\tilde{\boldsymbol{w}}_q^H\tilde{\boldsymbol{x}}(k). \tag{3.2}$$

where

$$\tilde{\boldsymbol{w}}_q=\left(\boldsymbol{w}_q^T,\boldsymbol{w}_q^H\right)^T \tag{3.3}$$

and $\boldsymbol{w}_q$ is given by (2.4). Since

$$\boldsymbol{z}(k)=\boldsymbol{B}^H\boldsymbol{x}(k), \tag{3.4}$$

we have

$$\boldsymbol{z}^*(k)=(\boldsymbol{B}^H)^*\boldsymbol{x}^*(k). \tag{3.5}$$

Combining (3.4) and (3.5), we obtain

$$\tilde{\boldsymbol{z}}(k)=\tilde{\boldsymbol{B}}^H\tilde{\boldsymbol{x}}(k), \tag{3.6}$$

where

$$\tilde{\boldsymbol{z}}(k)=\begin{pmatrix}\boldsymbol{z}(k)\\ \boldsymbol{z}^*(k)\end{pmatrix},\quad \tilde{\boldsymbol{B}}=\begin{pmatrix}\boldsymbol{B} & \boldsymbol{0}_{M\times(M-L)}\\ \boldsymbol{0}_{M\times(M-L)} & \boldsymbol{B}^*\end{pmatrix}. \tag{3.7}$$

Then the output of the branch via the adaptive weight $\tilde{\boldsymbol{w}}_a$ can be expressed by

$$\tilde{y}_b(k)=\tilde{\boldsymbol{w}}_a^H\tilde{\boldsymbol{z}}(k)=(\tilde{\boldsymbol{B}}\tilde{\boldsymbol{w}}_a)^H\tilde{\boldsymbol{x}}(k). \tag{3.8}$$

Combing (3.2) and (3.8), we obtain the output of WL GSC as follows

$$\tilde{y}_p(k)=\tilde{y}_c(k)-\tilde{y}_b(k)=(\tilde{\boldsymbol{w}}_q-\tilde{\boldsymbol{B}}\tilde{\boldsymbol{w}}_a)^H\tilde{\boldsymbol{x}}(k). \tag{3.9}$$

The structure of the proposed widely linear GSC beamformer is described by Fig. 1.

For the proposed WL GSC beamformer, the adaptive weight $\tilde{\boldsymbol{w}}_a$ is determined by minimizing

$$\min\ E\left[|\tilde{y}_p(k)|^2\right]. \tag{3.10}$$

The solution of (3.10) can be written as

$$\tilde{\boldsymbol{w}}_a=\left(\tilde{\boldsymbol{R}}_{\tilde{z}}\right)^{-1}\tilde{\boldsymbol{p}}_{\tilde{z}}, \tag{3.11}$$

Here, the $2(M-L)\times 2(M-L)$ matrix $\tilde{\boldsymbol{R}}_{\tilde{z}}=E[\tilde{\boldsymbol{z}}(k)\tilde{\boldsymbol{z}}^H(k)]$ is the covariance matrix of $\tilde{\boldsymbol{z}}(k)$ and $\tilde{\boldsymbol{p}}_{\tilde{z}}=E[\tilde{y}_c^*(k)\tilde{\boldsymbol{z}}(k)]$ denotes the $2(M-L)\times 1$ cross-correlation vector with $\tilde{\boldsymbol{z}}(k)$ and $\tilde{y}_c(k)$.

Many algorithms can be used to update $\tilde{\boldsymbol{w}}_a$ in (3.11) recursively, such as the least mean square (LMS) [1], normalised LMS (NLMS) [15], conjugate gradient (CG) [16] and recursive least squares (RLS) [1]. We choose the RLS algorithm to update $\tilde{\boldsymbol{w}}_a$ for its fast convergence speed in this paper. The estimation of covariance matrix $\tilde{\boldsymbol{R}}_{\tilde{z}}$ at time k is given by