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An automatically paired two-dimensional direction-of-arrival estimation method for two parallel uniform linear arrays

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ABSTRACT

In two-dimensional (2-D) direction-of-arrival (DOA) estimation, paring the azimuth and elevation angles of multiple sources is an important issue. In this letter, we propose a new automatically paired 2-D DOA estimation method by designing the geometry of two antenna subarrays and using the propagator method (PM). A special geometry between two parallel uniform linear arrays (ULAs) with a position displacement on the axial direction is proposed to facilitate the elevation and azimuth pairing and estimation. The simulation results have shown that the proposed method can achieve the same 2-D DOA estimation performance as the existing methods, while the complexity is reduced considerably.

1. Introduction

The direction-of-arrival (DOA) estimation is one of the important problems in many fields, such as wireless communication, radar, and sonar [1–5]. In recent years, a number of methods have been widely used in DOA estimation, including multiple signal classification (MUSIC) [6–9], estimation of signal parameters via rotational invariance techniques (ESPRIT) [10–12] and the propagator method (PM) [12–14]. Two-dimensional (2-D) DOA estimation involves the estimation of both the azimuth and elevation angles, which provide the full information of signal direction in three-dimensional space [15–18]. One important issue of 2-D DOA estimation is the joint and paired estimation of the azimuth and elevation angles. In [14], a PM-based 2-D DOA estimation method has been proposed for two parallel uniform linear arrays (ULAs), but it needs an extra pairing match progress to pair the azimuth and elevation angles. Ref. [19] has proposed an improved 2-D DOA estimation method for two parallel ULAs by using PM, and it uses the relation of submatrices to indirectly pair the azimuth and elevation angles to reduce the complexity, but it still needs some extra computation to obtain the estimate of the final paired angles. Ref. [20] has proposed a 2-D DOA estimation method by using ESPRIT for two shifted parallel ULAs and directly pairing the azimuth and elevation angles, but it has higher complexity and can be only used with a specific array geometry.

In this letter, we propose a new automatically paired 2-D DOA estimation method by using PM. The arrays structure in this letter is two parallel ULAs with a position displacement. The direction matrices of the arrays structure can be decomposed to produce two special diagonal submatrices which can be used to directly pair the azimuth and elevation angles and thus reduce the estimation complexity. By using the Euler's formula and adding the diagonal matrices together, we can obtain a new diagonal matrix whose diagonal elements contains the paired azimuth and elevation angles in both the magnitude and phase terms. Then the PM and the eigenvalue decomposition are used to estimate the paired 2-D DOA.

The rest of this letter is organized as follows. The system model is reviewed in Section 2. In Section 3, the details of the proposed method are described. Section 4 gives the simulation results, and Section 5 concludes the paper.

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2. System model

It is assumed that there are K far-field narrowband radio frequency sources whose radio waves impinge on the planar arrays as shown in Fig. 1. Two parallel ULAs are located in the X-Y plane with the array structure shown in Fig. 2. The spacing between two parallel ULAs is d_y, and the interelement spacing of each ULA is d_x, which satisfies d_x=2d_y. Unlike the parallel ULAs structure considered in [14,19,20], the two parallel ULAs have a position displacement d_c on the Y-coordinate. The position coordinates of elements in the first array are $(d_x \times i, 0), 0 \leqslant i < M$, while they are $(d_x \times i - d_c, d_y), 0 \leqslant i < M+1$, in the second array.

Suppose that θ_k and ϕ_k are the elevation and azimuth angles of the kth source. Then the output of the two parallel ULAs can be written as:

$$\mathbf{x}_1(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T = \mathbf{A}_1\mathbf{s}(t) + \mathbf{n}_1(t), \quad t = 1, 2, \ldots, N, \tag{1}$$

$$\mathbf{x}_2(t) = [x_{M+1}(t), x_{M+2}(t), \ldots, x_{2M+1}(t)]^T = \mathbf{A}_2\mathbf{\Phi}_2\mathbf{s}(t) + \mathbf{n}_2(t), \quad t = 1, 2, \ldots, N, \tag{2}$$

where N is the number of snapshots, $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T, s_k(t)$ is the kth source signal and the number of sources (i.e. K) is assumed as a known priori. $\mathbf{A}_1 = [\mathbf{a}_M(\theta_1, \phi_1), \mathbf{a}_M(\theta_2, \phi_2), \ldots, \mathbf{a}_M(\theta_K, \phi_K)], \mathbf{A}_2 = [\mathbf{a}_{M+1}(\theta_1, \phi_1), \mathbf{a}_{M+1}(\theta_2, \phi_2), \ldots, \mathbf{a}_{M+1}(\theta_K, \phi_K)]$, $\mathbf{a}_M(\theta_k, \phi_k) = [1, \exp(-j2\pi \frac{d_x}{\lambda}\alpha_k), \exp(-j2\pi \frac{2d_x}{\lambda}\alpha_k), \ldots, \exp(-j2\pi \frac{(M-1)d_x}{\lambda}\alpha_k)]^T$, and $\mathbf{a}_{M+1}(\theta_k, \phi_k) = [1, \exp(-j2\pi \frac{d_x}{\lambda}\alpha_k), \exp(-j2\pi \frac{2d_x}{\lambda}\alpha_k), \ldots, \exp(-j2\pi \frac{(M-1)d_x}{\lambda}\alpha_k), \exp(-j2\pi \frac{Md_x}{\lambda}\alpha_k)]^T$, where $\exp(\cdot)$ is the natural exponential function, $\alpha_k = \cos\phi_k \sin\theta_k$, λ is the wavelength of the source signal and $d_x \leqslant \lambda/2$, is assumed. $\mathbf{\Phi}_2$ is the diagonal matrix and $\mathbf{\Phi}_2 = \mathrm{diag}\Big[\exp\Big(-j2\pi(\frac{d_y}{\lambda}\beta_1 - \frac{d_c}{\lambda}\alpha_1)\Big), \exp\Big(-j2\pi(\frac{d_y}{\lambda}\beta_2 - \frac{d_c}{\lambda}\alpha_2)\Big), \ldots, \exp\Big(-j2\pi(\frac{d_y}{\lambda}\beta_K - \frac{d_c}{\lambda}\alpha_K)\Big)\Big]$, where $\beta_k = \sin\phi_k \sin\theta_k$. $\mathbf{n}_1(t)$ and $\mathbf{n}_2(t)$ are the white Gaussian noise with zero means and covariance matrices $\sigma^2\mathbf{I}_M$ and $\sigma^2\mathbf{I}_{M+1}$, respectively. Here $\mathbf{I}_M$ is the identity matrix of size M. It is assumed that the K sources signals are uncorrelated with each other and noise.

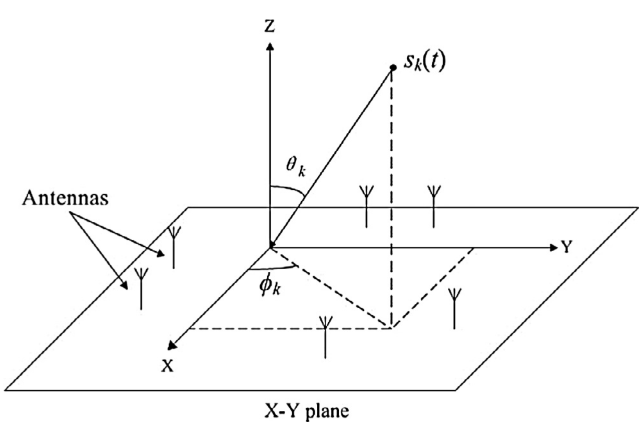

Fig. 1. Coordinate system for 2-D DOA estimation.

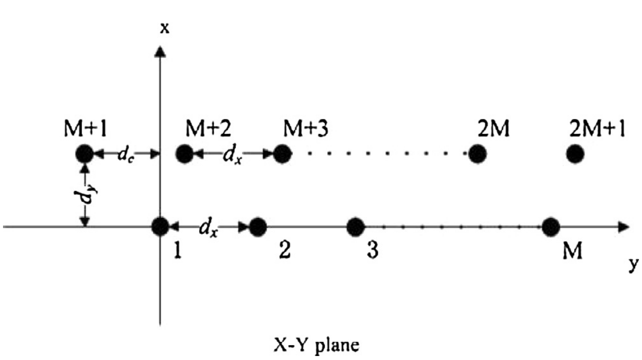

Fig. 2. Illustration of the array geometry.

3. 2-D auto-paired angle estimation method

3.1. Auto-paired DOA estimation using PM

Let $\mathbf{X}_{21}(t)$ and $\mathbf{X}_{22}(t)$ be two vectors which contain the first M components and the last M components of $\mathbf{X}_2(t)$ respectively. That is, $\mathbf{X}_{21}(t) = [x_{M+1}(t), x_{M+2}(t), \ldots, x_{2M}(t)]^T, \mathbf{X}_{22}(t) = [x_{M+2}(t), x_{M+3}(t), \ldots, x_{2M+1}(t)]^T$. It is clear that $\mathbf{X}_{21}(t)$ and $\mathbf{X}_{22}(t)$ can be expressed as:

$$\mathbf{X}_{21}(t) = \mathbf{A}_1\mathbf{\Phi}_2\mathbf{s}(t) + \mathbf{n}_{21}(t), \tag{3}$$

$$\mathbf{X}_{22}(t) = \mathbf{A}_1\mathbf{\Phi}_3\mathbf{\Phi}_2\mathbf{s}(t) + \mathbf{n}_{22}(t) = \mathbf{A}_1\mathbf{\Phi}_1\mathbf{s}(t) + \mathbf{n}_{22}(t), \tag{4}$$

where $\mathbf{n}_{21}$ and $\mathbf{n}_{22}$ are the white Gaussian noise vectors of zero means and covariance matrix $\sigma^2\mathbf{I}_M$, and $\mathbf{\Phi}_1 = \mathrm{diag}\Big[\exp\Big(-j2\pi(\frac{d_y}{\lambda}\beta_1 + \frac{d_x - d_c}{\lambda}\alpha_1)\Big), \exp\Big(-j2\pi(\frac{d_y}{\lambda}\beta_2 + \frac{d_x - d_c}{\lambda}\alpha_2)\Big), \ldots, \exp\Big(-j2\pi(\frac{d_y}{\lambda}\beta_K + \frac{d_x - d_c}{\lambda}\alpha_K)\Big)\Big], \mathbf{\Phi}_3 = \mathrm{diag}\big[exp(-j2\pi\frac{d_x}{\lambda}\alpha_1), exp(-j2\pi\frac{d_x}{\lambda}\alpha_2), \ldots, exp(-j2\pi\frac{d_x}{\lambda}\alpha_K)\big]$.

Now define the array out as:

$$\mathbf{Y}(t) = \begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_{21}(t) + \mathbf{x}_{22}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_1(\mathbf{\Phi}_1 + \mathbf{\Phi}_2) \end{bmatrix}\mathbf{s}(t) + \begin{bmatrix} \mathbf{n}_1(t) \\ \mathbf{n}_{21}(t) + \mathbf{n}_{22}(t) \end{bmatrix} = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{5}$$

where $\mathbf{A} \in \mathbb{C}^{2M \times K}, \mathbf{A}_1 \in \mathbb{C}^{M \times K}, \mathbf{\Phi}_1 + \mathbf{\Phi}_2 \in \mathbb{C}^{K \times K}$. Then the estimation of covariance matrix of the array output can be written as:

$$\mathbf{R}_Y = \frac{1}{N}\sum_{t=1}^{N}\mathbf{Y}(t)\mathbf{Y}^H(t) = [\mathbf{R}_1 \quad \mathbf{R}_2], \tag{6}$$

where $\mathbf{R}_1 \in \mathbb{C}^{2M \times K}, \mathbf{R}_2 \in \mathbb{C}^{2M \times (2M-K)}, \mathbf{R}_1$ and $\mathbf{R}_2$ are the submatrices of $\mathbf{R}_Y$. The propagator matrix $\mathbf{P} \in \mathbb{C}^{K \times (2M-K)}$, can be estimated by the minimization problem as [19]:

$$\mathbf{P} = (\mathbf{R}_1^H\mathbf{R}_1)^{-1}\mathbf{R}_1^H\mathbf{R}_2, \tag{7}$$

Define the matrix $\mathbf{P}_X \in \mathbb{C}^{2M \times K}$ as:

$$\mathbf{P}_X = \begin{bmatrix} \mathbf{I}_{K \times K} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \end{bmatrix}, \quad \mathbf{P}_1 \in \mathbb{C}^{M \times K}, \mathbf{P}_2 \in \mathbb{C}^{M \times K}, \tag{8}$$

Let $\mathbf{A}_c \in \mathbb{C}^{K \times K}$ be the first K rows of $\mathbf{A}, \mathbf{A}_d \in \mathbb{C}^{(2M-K) \times K}$ is the other $2M-K$ rows of $\mathbf{A}$. According to the illustration of $\mathbf{A}_1$, it is clear that $\mathbf{A}_c$ is a Vandermonde matrix and the determinant of $\mathbf{A}_c$ [21] is $\det(\mathbf{A}_c) = \prod_{1 \leqslant i < j \leqslant K}\big[exp(-j2\pi\frac{d_x}{\lambda}\alpha_i) - exp(-j2\pi\frac{d_x}{\lambda}\alpha_j)\big]$. Since it is virtually impossible that the direction of one source, (ϕ_i, θ_i), is exactly the same as that of another source, (ϕ_j, θ_j), and $\alpha_k = \cos\phi_k \sin\theta_k$, it can be generally assumed that $\alpha_i \neq \alpha_j$, and $\det(\mathbf{A}_c) \neq 0$. Therefore, $\mathbf{A}_c$ is assumed as a non-singular matrix [12,14,19]. For the propagator matrix $\mathbf{P}$, it has [19]:

$$\mathbf{P}^H\mathbf{A}_c = \mathbf{A}_d, \tag{9}$$

Using (5), (8) and (9) produces:

$$\mathbf{P}_X\mathbf{A}_c = \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \end{bmatrix}\mathbf{A}_c = \begin{bmatrix} \mathbf{I}_{K \times K} \\ \mathbf{P}^H \end{bmatrix}\mathbf{A}_c = \begin{bmatrix} \mathbf{A}_c \\ \mathbf{A}_d \end{bmatrix} = \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_1(\mathbf{\Phi}_1 + \mathbf{\Phi}_2) \end{bmatrix}, \tag{10}$$