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An evolutionary algorithm for multi-criteria inverse optimal value problems using a bilevel optimization model

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ABSTRACT

Given a linear program, a desired optimal objective value, and a set of feasible cost vectors, one needs to determine a cost vector of the linear program such that the corresponding optimal objective value is closest to the desired value. The problem is always known as a standard inverse optimal value problem. When multiple criteria are adopted to determine cost vectors, a multi-criteria inverse optimal value problem arises, which is more general than the standard case. This paper focuses on the algorithmic approach for this class of problems, and develops an evolutionary algorithm based on a dynamic weighted aggregation method. First, the original problem is converted into a bilevel program with multiple upper level objectives, in which the lower level problem is a linear program for each fixed cost vector. In addition, the potential bases of the lower level program are encoded as chromosomes, and the weighted sum of the upper level objectives is taken as a new optimization function, by which some potential nondominated solutions can be generated. In the design of the evolutionary algorithm some specified characteristics of the problem are well utilized, such as the optimality conditions. Some preliminary computational experiments are reported, which demonstrates that the proposed algorithm is efficient and robust.

1. Introduction

General optimization problems are to determine the values of variables such that the objectives can achieve the optima on the condition that all coefficients are given in objectives and constraints. Inverse optimization is to execute an 'inverse' procedure, that is to infer some parameters in objective function or in constraints such that the optimal solutions satisfy a pre-specified standard. A standard inverse optimization problem can be described as follows: given an optimization problem $P: \min_{x \in X} c^T x$ and a desired optimal solution $x \in X$, determine a cost vector c such that x is optimal to problem P. In addition, some additional conditions on c are often taken into account. For example, c should be as close to a pre-determined vector c' as possible under some ℓ_p-norm, that is, the deviation $\|c - c'\|_p$ should be minimized. This problem was first introduced by Burton and Toint [1,2] in a shortest path problem, in which all weights on edges need to be determined such that some pre-specified edges can form the short-path. In this class of problems, the objective function is also decided by the selected measure norm. For instance, when the deviation is evaluated by ℓ_2-norm, the objective function of inverse optimization can be transformed into a convex quadratic function [1]. If P is a linear program, and ℓ_1 and ℓ_∞ norms are adopted, linear programming can be utilized to deal with this class of problems [3,4]. For a comprehensive survey of the literature on inverse optimization, please refer to [5].

If the problem is to find a vector c such that the objective value is equal or close to a pre-specified value, the problem is known as an inverse optimal value problem. Obviously, the inverse optimal value problem is a generalized version of the standard inverse optimization problem, because the objective values must be close to each other when two optimal solutions approach for any continuous functions. This kind of problems can be stated as follows: given an optimization problem P, a desired optimal objective value z^*, and a set C of feasible cost vectors, determine a cost vector $c^* \in C$ such that the corresponding optimal objective value of P is closest to z^* [6,7]. The inverse optimal value problem has received little attention in the literature. Berman [8] discussed a minimal location model, which is to determine edge lengths in a graph such that the induced minimal distance from a given vertex to all other vertices is within prescribed bounds, and shows the problem is NP-complete for general graphs.

This paper is motivated by an application in telecommunication bandwidth pricing proposed by Paleologo and Takriti [9]. In this research, the problem the authors consider is to infer the correct

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prices on city-to-city links on its bandwidth network, such that the total price of the links on the cheapest path between arbitrary two cities should be close to the pre-specified one. The problem is modeled as a directed graph with m nodes and n arcs, in which the length associated to each arc is the unknown price. Mathematically, the problem is to determine the length c_j of each arc j, $j = 1, \ldots, n$, so that the length of the shortest path between a node pair is equal to a given value. We denote by K the number of origin-destination pairs for which the desired shortest distance p_k is known. The problem becomes that of finding a nonnegative c such that

$$\begin{cases} \min\limits_{x_k} c^T x_k = p_k, \\ \text{s.t. } \bar{A}x_k = b_k, x_k \geq 0. \\ k = 1, \ldots, K \end{cases} \tag{1}$$

Alternatively, we may formulate the problem as a multi-criteria optimization problem

$$\min_{c \geq 0}(f_1(c), \ldots, f_K(c)) \tag{2}$$

Here, $f_i(c) = |\min\limits_{x_i \in X_i} c^T x_i - p_i|$, $X_i = \{x_i | \bar{A}x_i = b_i, x_i \geq 0\}$, $i = 1, \ldots, K$, C is a feasible region and $|.|$ denotes the absolute value. In fact, the bandwidth pricing problem is essentially a multi-criteria inverse optimal value problem [7,9].

For the case in which $K = 1$ is a trivial one, Ahmed and Guan [7] showed that the inverse optimal value problem is NP-hard in general, and for the case that the set of feasible cost vectors is polyhedral, the authors developed an algorithm for the problem based on solving linear and bilinear programming problems. Lv [6] proposed a bilevel programming model and applied the duality principle of linear program to transform the problem into a single level nonlinear program. Then, a penalty function method was used to solve the equivalent problem. Despite the fact that these procedures are efficient for the trivial case (single-objective inverse optimal value problem), they cannot be extended directly to multi-criteria cases. From the multi-objective point of view, Paleologo and Takriti [9] suggested a mixed-integer programming formulation by the weighted sum of the objectives for this bandwidth pricing problem. However, it can only give one optimal solution in each run of the algorithm and needs to solve a mixed-integer program which is computationally intractable.

In fact, problem (2) is a special multi-criteria bilevel programming problem (MCBLPP). MCBLPP describes a hierarchical structure, the constraint region of the first level problem (leader's problem/upper level problem) is implicitly determined by the second level optimization problem (follower's problem/lower level problem), and the upper and/or lower levels have more than one objective. At present, some algorithmic approaches and theoretical results have been developed for this kind of the problems [10–13]. In optimization procedures involving multiple objectives, some population-based algorithms have been widely adopted since these algorithms can provide a Pareto optimal front for various preferences, such as evolutionary algorithm (EA) and it's variations. Deb [13] proposed an efficient EA for solving MCBLPPs, which almost puts no requirements on all functions involved, such as convex or differentiable. But for problem (2) in which the lower level is single objective and is also linear for any fixed cost vector c, these procedures are computation-expensive.

The purpose of this paper is to discuss a special MCBLPP which is a generalized version of problem (2) and then provide an efficient approach for solving most of inverse optimization problems with multiple criteria. Based on the proposed MCBLPP model, we develop an efficient EA by taking advantage of a dynamic weighted aggregation method as well as the optimality conditions of the optimization problem. In our approach, the potential bases of the lower level program are encoded as chromosomes, and the weighted sum of the upper level objectives is taken as a new optimization function, by which some potential nondominated solutions can be generated. It is worth noting that the coding scheme makes the algorithm evidently different from other EAs, such as Deb's method using NSGA-II, mainly because the coding technique makes the search space become finite even for a continuous MCBLPP.

This paper is organized as follows. Multi-criteria bilevel programming model is proposed and some basic notations are presented in Section 2, and Section 3 gives evolutionary operators and displays our algorithm. Some computational examples are given and solved in Section 4. We finally conclude our paper in Section 5.

2. Multi-criteria bilevel programming model

Problem (2) can be rewritten as

$$\begin{cases} \min\limits_{c \geq 0}(|c^T x_1 - p_1|, \ldots, |c^T x_K - p_K|) \\ \min c^T x_1 \\ \text{s.t. } \bar{A}x_1 = b_1, x_1 \geq 0; \\ \min c^T x_2 \\ \text{s.t. } \bar{A}x_2 = b_2, x_2 \geq 0; \\ \ldots \\ \min c^T x_K \\ \text{s.t. } \bar{A}x_K = b_K, x_K \geq 0. \end{cases} \tag{3}$$

Obviously, the problem is a multi-criteria bilevel programming problem with multiple lower level problems. Note that no common variables are shared between arbitrary two lower level problems, therefore it is equivalent to

$$\begin{cases} \min\limits_{c \geq 0}(|c^T x_1 - p_1|, \ldots, |c^T x_K - p_K|) \\ \min \sum\limits_{i=1}^{K} c^T x_i \\ \text{s.t. } \bar{A}x_i = b_i, x_i \geq 0, i = 1, \ldots, K. \end{cases} \tag{4}$$

Further, set

$$c^1 = (\underbrace{c^T, 0, \ldots, 0}_{K})$$

$$\ldots$$

$$c^K = (\underbrace{0, \ldots, 0, c^T}_{K})$$

and

$$\bar{x} = (x_1^T, \ldots, x_K^T)^T, C = c^1 + \cdots + c^k$$

Then, (4) can be reformed as

$$\begin{cases} \min\limits_{c \geq 0}(|c^1\bar{x} - p_1|, \ldots, |c^K\bar{x} - p_K|) \\ \min C\bar{x} \\ \text{s.t. } \operatorname{diag}(\bar{A})\bar{x} = b, \bar{x} \geq 0 \end{cases} \tag{5}$$