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Super resolution image reconstruction using weighted combined Pseudo-Zernike moment invariants

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ABSTRACT

This paper proposes a multi-image super resolution reconstruction method to estimate a high resolution (HR) image without performing exact image registration, de-blurring and de-noising of available low resolution images. Pixels in the HR image are estimated based on the weighted average of the neighborhood pixels. The weights in the averaging process measure the correlation between pixels and are calculated using a set of feature vectors based on weighted combined Pseudo-Zernike moment invariants (WCPZMIs) of optimum order. WCPZMIs are those reliable Pseudo-Zernike moment invariants (PZMIs) which are simultaneously insensitive to geometric transformations (rotation, scaling, and translation) as well as degradations (blur) and are relatively weighted according to their reconstruction capability. An energy minimization scheme is employed to select the optimal order of WCPZMIs which makes a trade-off between the quality of reconstruction and robustness to noise. An efficient way of weighting the feature vectors and the recursive approach for the computation of PZMIs are utilized to reduce the computational overload of the reconstruction process. Besides, an appropriate square-to-circle mapping followed by a radial geometric moment-to-PZM approximation is adopted to reduce the geometric and the numerical error respectively. Experimental results of the proposed method outperform as compared to similar contributions in literature.

1. Introduction

Super resolution reconstruction (SRR) is one of the low cost, software level alternative to reconstruct a high resolution (HR) image from multiple numbers of low resolution (LR) images. In the image SRR problem, the imaging system plays a vital role as it maps the basic relationship between the original image and the observed images. Estimation of a HR image back from a sequence of LR images follows three basic steps such as image registration, de-blurring, de-noising and up-sampling of the observed LR images to produce the HR image and basically ill-posed in nature.

Particularly, in multi image SRR problem, image registration prior to reconstruction is essential in order to utilize the non redundant information contained in the LR images. The lesser is the registration error, the more is the accuracy of reconstruction process. However, existing methods of image registration are strongly influenced by the accurate motion estimation and till now it is an open challenge to obtain an accurate image registration scheme. As an alternative to reduce this difficulty, recently the fuzzy registration schemes [1–3]are utilized to perform image registration and image reconstruction process simultaneously. In [1], a non-local means based SRR (NLM-SR) method is proposed where the image registration along with image reconstruction process is achieved by a neighborhood averaging process. However, in the real scenario the LR images are affected by several image degradations and hence finding the appropriate correlation between images becomes difficult. On the other hand, SRR methods described in [2,3] utilize orthogonal moment invariant feature vectors for the weight calculation as these features are insensitive to different types of degradation present in LR images. In [2], Gao et al. employed the feature vectors based on the magnitude of the Zernike moment (ZM-SR) invariants whereas in [3], Kanan et al. utilized the feature vectors based on the magnitude and phase of the Pseudo Zernike moment invariant (PZM-Mag-SR and PZM-Phase-SR). However, the above mentioned moment based methods work well when the LR images are exclusively rotated version of the target image. But, in a complex imaging system where the LR images are corrupted by the geometric transformations as well

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as the blur degradation, the SRR methods produce poor results. Besides, the SRR method in [2,3] suffer from numerical error and is computationally heavy due to the direct method for the computation of moment invariants. To address these shortcomings, this paper proposes a computationally efficient SRR method. The proposed SRR method utilizes weighted averaging of the neighborhood pixels of the LR images to produce each and every pixel of the HR image.The feature vectors of the image are based on the weighted combined pseudo zernike moment invariants (WCPZMIs), which are simultaneously invariant to both geometric transformations as well as blur degradation and are free from numerical error.

The rest of the discussion follows: Section 2 provides the formulation and description of the proposed work. Performance analysis of the proposed method with other similar type of works in literature is presented in Section 3. Section 4 provides the conclusion of the paper with some of its future aspects.

2. Proposed SRR method

2.1. Pseudo Zernike Moment (PZM) of image

In this work, an appropriate mapping formulation is used to map the whole intensity image $f(a,b)$ of size $N \times N$ into $f(u_i, v_j)$ on the unit circle to minimize the geometrical error [4]. The mapping scheme is described in Eq. (1).

$$\left.\begin{aligned} &\{f(a,b)\}_{N\times N} \rightarrow \{f(u_i, v_j)\}_{[-1,1]\times[-1,1]} \\ &u_i = c_1(i+0.5)+c_2 i = 0,1,\ldots\ldots.(N-1) \\ &v_j = c_1(j+0.5)+c_2 j = 0,1,\ldots\ldots..(N-1) \\ &c_1 = \tfrac{\sqrt{2}}{N-1};\; c_2 = -\tfrac{1}{\sqrt{2}} \end{aligned}\right\} \tag{1}$$

Eq. (2), provides the formulation for the calculation of PZMs [5] of image $f(u_i, v_j)$ of order m with n repetition.

$$PZM^f_{(m,n)} = \frac{(m+1)}{\pi}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} f(u_i, v_j)\int_{u_i-\frac{\Delta u}{2}}^{u_i+\frac{\Delta u}{2}}\int_{v_j-\frac{\Delta v}{2}}^{v_j+\frac{\Delta v}{2}} V^*_{m,n}(a,b)\,da\,db \tag{2}$$

The superscript f is used for image $f(u_i, v_j)$. $V^*_{m,n}$ is the complex conjugate of the pseudo zernike polynomials [6] and is presented in Eq. (3).

$$V_{m,n} = \mathrm{R}_{m,n}(r)\,e^{jn\theta} \tag{3}$$

where $r = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}(b/a)$. $\mathrm{R}_{m,n}(r)$ denotes the pseudo zernike radial polynomial and is described in Eq. (4).

$$\begin{aligned} \mathrm{R}_{m,n}(r) &= \sum_{t=0}^{m-|n|} S_{m|n|k}\, r^{(m-t)} \\ &= \sum_{t=0}^{m-|n|}(-1)^t \frac{(2m+1-t)!}{t!(m-|n|-t)!(m+|n|+1-t)!} r^{(m-t)} \end{aligned} \tag{4}$$

2.2. Creation of feature vectors based on CPZMIs of image $f(u_i, v_j)$

This subsection aims at creating a set of feature vectors based on PZMs which are combinedly invariant to geometric transformations and blur and are termed as combined pseudo zernike moment invariants (CPZMIs). Formulation for the CPZMIs follows the same formulation as described in [7] and is given in Eq. (5).

$$CPZMIs^f_{(q+k,q)} = G^f_{(q+k,q)} - \frac{1}{G^f_{00}\pi}\sum_{i=0}^{k-1} IB^f_{(q+i,q)}\sum_{j=0}^{k-i} G^f_{(j,0)} A(q,k,i,j) \tag{5}$$

where $CPZMIs^f_{(q+k,q)}$ is the $(q+k)$th order CPZMIs of image $f(u_i, v_j)$. $G_{(q+k,q)}$ denotes the $(q+k)$th order geometric transformation invariants of the image $f(u_i, v_j)$. $IB_{(q+k,q)}$ is $(q+k)$th order blur invariants [7]. The next section describes about the minimization of numerical error and uses an exact and computationally inexpensive approach in the calculation of PZMs to reduce computational overload.

2.3. Minimization of numerical error and reduction of computational time

Numerical error is caused due to the zeroth order approximation of the double integration in the PZM calculation process (Eq. (2)). This error is eradicated by utilizing the radial geometric moment-to-PZM approximation and Eq. (2) is replaced by Eq. (6).

$$PZM^f_{(m,n)} = \frac{(m+1)}{\pi}\left[\begin{array}{l} \left\{\sum_{t=n;}^{m} S_{m|n|k}\sum_{j=0}^{q}\sum_{s=0}^{n}(-i)^s\binom{q}{j}\binom{n}{s} Gm_{(k-2j-s,2j+s)}\right\}_{t-n=even} + \\ \left\{\sum_{t=n+1;}^{m} S_{m|n|k}\sum_{j=0}^{p}\sum_{s=0}^{n}(-i)^s\binom{p}{j}\binom{n}{s} RGm_{(k-2j-s-1,2j+s)}\right\}_{t-n=odd} \end{array}\right] \tag{6}$$

where $q = t-n$; $p = (t-n-1)/2$ and $-i = \sqrt{(-1)}$. $Gm_{(p,q)}$ and $RGm_{(p,q)}$ define the geometric and radial geometric moments. The formulation of $Gm_{(p,q)}$ and $RGm_{(p,q)}$ for image $f(u_i, v_j)$ are defined in Eqs. (7) and (8) respectively.

$$\begin{aligned} Gm_{(p,q)} = &\frac{1}{(p+1)(q+1)}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} f(u_i, v_j)\left[\left(u_i+\frac{\Delta u_i}{2}\right)^{p+1}\right. \\ &\left.-\left(u_i-\frac{\Delta u_i}{2}\right)^{p+1}\right]\left[\left(v_i+\frac{\Delta v_i}{2}\right)^{q+1}-\left(v_i-\frac{\Delta v_i}{2}\right)^{q+1}\right] \end{aligned} \tag{7}$$

$$\begin{aligned} RGm_{(p,q)} = &\frac{1}{(p+1)(q+1)}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}(u_i+v_j)^{\frac{1}{2}} f(u_i, v_j)\left[\left(u_i+\frac{\Delta u_i}{2}\right)^{p+1}\right. \\ &\left.-\left(u_i-\frac{\Delta u_i}{2}\right)^{p+1}\right]\left[\left(v_i+\frac{\Delta v_i}{2}\right)^{q+1}-\left(v_i-\frac{\Delta v_i}{2}\right)^{q+1}\right] \end{aligned} \tag{8}$$

From the analysis, it is found that Eqs. (7) and (8) follow simple mathematical integration rules to find the exact computation PZMs which removes the numerical integration error. Eq. (9) provides the recursive approach as proposed in [8] to calculate the coefficients of $S_{m|n|k}$ to reduce the computational time of the process.

$$\begin{aligned} &S_{mmm} = 1;\quad S_{m(n-2)m} = \frac{(m+n)}{(m-n+2)} S_{mnm}; \\ &S_{mn(k-2)} = -\frac{(k-n)(k+n)}{(m+k)(m-k+2)} S_{mnk} \end{aligned} \tag{9}$$

2.4. Image SRR using neighborhood weighted averaging process

The proposed SRR method utilizes a local and patch wise neighborhood weighted averaging process for estimating the pixel value in the HR image. Eq. (10) provides the formulation for the estimation of (p, q) pixel value of the HR image $\hat{X}_{HR}$;

$$\hat{X}_{HR}(p,q) = \frac{\sum_{i\in[1,..n]}\sum_{(x,y)\in N(p,q)} W_{PZM}(p,q,x,y,i)\, y_i(x,y)}{\sum_{i\in[1,..n]}\sum_{(x,y)\in N(p,q)} W_{PZM}(p,q,x,y,i)} \tag{10}$$

where $\hat{X}_{HR}$ is the estimated HR image. i is the number of LR images to be utilized in the reconstruction process. $N(p,q)$ denotes the neighborhood of (p,q) pixel. $y_i(x,y)$ denotes the ith LR image