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Convergence and steady state analysis of a tap-length optimization algorithm for linear adaptive filters [☆]

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ABSTRACT

An adaptive filter with a large number of coefficients or taps results in slow convergence and increases the computational load. To overcome this problem, optimum tap-length selection algorithms for automatic structure adaptation in linear adaptive filters have been proposed, which provide improved convergence rate without degrading the steady state performance. The most recent variable-tap length, variable step normalized least mean square algorithm with variable error spacing (VT-VSNLMS_{VE}), employs a sliding window weight update and achieves better results in reducing the structural as well as computational complexity compared to its predecessors. But it does not present a convergence and steady-state analysis of the proposed algorithm. In the present paper, we have made a convergence and steady state analysis of the VT-VSNLMS_{VE} algorithm. A mathematical formulation of the variable step-size, mean square equations and steady state tap-length is obtained that provides an idea regarding the applicability of the variable tap-length algorithm for many applications using higher-order adaptive filters. Computer simulations are presented in support of the algorithm analysis under predefined assumptions.

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1. Introduction

The total number of weights in an adaptive filter is known as the filter length or tap-length. It is a key parameter that affects the convergence of adaptive algorithms [1]. In system identification applications, adaptive filters are employed for acoustic echo cancellation, feedback cancellation, noise compensation, etc. However, in certain specific applications like acoustic echo cancellation there is a need of hundreds to thousands of filter weights to model the unknown impulse response [2]. An improper selection of the total number of filter coefficients may result in underestimated filter length that introduces extra error, while an overestimated filter length brings extra computational burden [3]. Thus, the tap-length optimization algorithms have received much attention. These algorithms try to find the dynamically-varying filter order with faster convergence rate and high accuracy [4]. The tap-length adaptation is carried out until the coefficient requirement remains unchanged with further increase in iterations and the unknown impulse response is identified [5].

Several tap-length optimization adaptive algorithms, such as the fractional tap-length least mean square algorithm (FT-LMS) [4], variable leakage factor based FT-LMS algorithm [6] and variable tap variable step LMS (VT-VSLMS) algorithm [7–9], have been presented to reduce the complexity of the adaptive designs and to improve the convergence. These existing algorithms bring their own limitations while selecting key parameters for tap-length adaptation, which have been highlighted in [10]. Similarly, the results concerning the convergence rate, steady state performance and minimum tap-length requirement vary from one variable tap-length algorithm to another. The variable tap-length, variable step normalized least mean square algorithm with variable error width (VT-VSNLMS_{VE}) presented in [10] tries to remove the downsides of the existing algorithms in both high and low noise scenarios. The structure adaptation algorithm proposed in [10] maintains a trade-off between the mean square error and convergence. This paper presented a sliding window weight update algorithm along with the tap-length adaptation algorithm to reduce the tapped delay line filter design complexity as well as computational complexity in a comparatively improved manner. The parameters for finding the optimum filter weights are obtained dynamically rather than set to a randomly predefined value as in the FT-LMS algorithm. But, the article in [10] does not provide a convergence

[☆] The first part of this paper is presented in [10].

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and steady state analysis for the proposed VT-VSNLMS_{VE} algorithm.

In this paper, a step size convergence and steady state performance of the VT-VSNLMS_{VE} tap-length optimization algorithm is derived. At first, the sliding window weight update equation is reviewed, which gives an insight to the step-size convergence. Then, based on the tap-length learning equation, a mathematical formulation for the mean square equations and steady state analysis are provided for linear FIR adaptive filters. The analysis and associated discussion give an idea about the practical usability of the variable tap-length algorithm. The rest of the paper is organized as follows. In Section 2, a brief description of the VT-VSNLMS_{VE} algorithm is presented. Section 3 presents the convergence analysis, mean square equations and the steady state analysis of the VT-VSNLMS_{VE} tap-length adaptation algorithm. A simulation study is provided in Section 4.

Throughout the paper, the following notations are adopted, $E[\cdot]$ for the expected value, $\min\{\cdot\}$ for the minimum of, $(\cdot)^T$ for the transpose of a vector or matrix, $\text{tr}\{\cdot\}$ for the trace operation, $\langle \cdot \rangle$ for converting a fractional value to the nearest integer, $\|\cdot\|$ norm of a vector, $\|\cdot\|_2$ for the l_2 norm, $\det[\cdot]$ for the determinant, $\text{diag}[\cdot]$ for the diagonal elements of a matrix.

2. The VT-VSNLMS_{VE} tap-length optimization algorithm

In the VT-VSNLMS_{VE} tap-length adaptation algorithm [10], the weight update is carried out using a sliding window technique as

$$\mathbf{w}_{L(n)}(n+K) = \mathbf{w}_{L(n)}(n) + \mu_{L(n)}(n) \sum_{i=n}^{n+K-1} e_{L(n)}(i) \mathbf{x}_{L(n)}(i), \quad (1)$$

where $L(n)$ is the instantaneous variable tap-length obtained from the tap-length optimization algorithm, $0 < K < L(n)$ is the sliding window frame size and $\mu_{L(n)}(n)$ is the variable step-size, which is expressed as

$$\mu_{L(n)}(n) = \frac{\mu'}{\sigma_{x,n+K-1}^2 (L(n) + 2)}, \quad (2)$$

where μ' is a constant [10], $\sigma_{x,n+K-1}^2$ is the variance of input signal with $n+K-1$ initial coefficients. The segmented error with respect to tap-length $L(n)$ is given by

$$e_K^L(n) = d(n) - y(n), \quad (3)$$

where

$$y(n) = \sum_{i=n}^{n+K-1} \mathbf{w}_{L(n)}(i) \mathbf{x}_{L(n)}(i) = \mathbf{w}_{L(n),1:K}(n) \mathbf{x}_{L(n),1:K}^T(n), \quad (4)$$

$$d(n) = \mathbf{w}_{L_{\text{opt}}}(n) \mathbf{x}_{L_{\text{opt}}}^T(n) + t(n), \quad (5)$$

$\mathbf{w}_{L(n),1:K}(n) = [w_1(n), w_2(n), \dots, w_{L(n)}(n)]^T$, $\mathbf{x}_{L(n),1:K}(n) = [x(n), x(n-1), \dots, x(n-L(n)+1)]^T$ with $1:K$ denoting the frame size of sliding window, L_{opt} is the optimum filter order, $\mathbf{w}_{L_{\text{opt}}}(n)$ and $\mathbf{x}_{L_{\text{opt}}}(n)$ are the weight vector and input vector respectively, pertaining to optimum tap-length and $t(n)$ is the zero mean white Gaussian additive noise.

The mean square segmented error is obtained as

$$Q_K^L(n) = E[(e_K^L(n))^2]. \quad (6)$$

The cost function used for finding the optimum tap-length in the VT-VSNLMS_{VE} algorithm is defined as the difference between the mean square error (MSE) at two different filter coefficients separated by error spacing Δ_L as

$$\min\{L|Q_{L-\Delta_L} - Q_L \leq \xi\}, \quad (7)$$

where ξ is a small positive threshold value, defined as the ratio of a positive constant Ψ_n that restricts the tap-length to attain a large value and the tap-length adaptation step-size $\bar{\Psi}_n$ [10]. Both these parameters are derived in [10]. Finally, the tap-length optimization algorithm is obtained as

$$L_f(n+1) = L_f(n) - \Psi_n + \varphi \bar{\Psi}_n Q_{\text{DF}}^L(n), \quad (8)$$

where $L_f(n)$ is the fractional tap-length, which is finally rounded to the nearest integer as $L(n) = \langle L_f(n) \rangle$, $\varphi = \text{sgn}(L_f(n) - L_f(n-1))$. Both the parameters are set as per the analysis made in [10] and the estimate of the difference of the mean square errors is

$$Q_{\text{DF}}^L(n) = (e_{K,L(n)}^L(n))^2 - (e_{K,L(n)-\Delta_L(n)}^L(n))^2. \quad (9)$$

3. The algorithm analysis

In this section, the variable tap-length, variable step normalized least mean square algorithm with variable error width, is analysed further by deriving the following:

- (i) The step-size convergence
- (ii) Mean square equations
- (iii) Steady-state performance

3.1. The step-size convergence

The weight update equation in the VT-VSNLMS_{VE} is based on a sliding window approach, where the frame size of each window is K and $1 < K < L(n)$. For convergence analysis, each frame can be considered as a sub-vector of the actual weight vector. If we consider P to be the number of sub-vectors each with a frame size K , then the desired signal in (5) can be modified as

$$d(n) = \sum_{j=1}^P \mathbf{w}_{j,L_{\text{opt}}} \mathbf{x}_{j,L_{\text{opt}}}^T(n) + t(n) \quad (10)$$

where $\mathbf{w}_{j,L_{\text{opt}}} = [w_{1,1}, w_{1,2}, \dots, w_{j,L_{\text{opt}}}]^T$ is the Wiener optimum solution of the j^{th} sub vector and $\mathbf{x}_{j,L_{\text{opt}}}(n) = [x(n - (k_1 + k_2 + \dots + k_j)), x(n - (k_1 + k_2 + \dots + k_{j+1})), \dots, x(n - (k_1 + k_2 + \dots + k_j + L_{\text{opt}} - 1))]^T$ is the input sequence for the sliding window approach. Similarly, the output of adaptive filter in (4) is represented as

$$y(n) = \sum_{j=1}^P \mathbf{w}_{j,L(n)}(n) \mathbf{x}_{j,L(n)}^T(n), \quad (11)$$

Also, the error signal in (3) can be rewritten as

$$e_K^L(n) = e_{j,L(n)} = d(n) - \sum_{j=1}^P \mathbf{w}_{j,L(n)}(n) \mathbf{x}_{j,L(n)}^T(n). \quad (12)$$

The weight adaptation equation in (1) can be presented in a modified form as

$$\mathbf{w}_{j,L(n)}(n+1) = \mathbf{w}_{j,L(n)}(n) + \mu_{j,L(n)}(n) \mathbf{x}_{j,L(n)}(n) e_{j,L(n)}(n). \quad (13)$$

For convergence analysis, let us define the weight error vector for the j^{th} sub-vector as

$$\zeta_j(n) = \mathbf{w}_{j,L(n)}(n) - \mathbf{w}_{j,L_{\text{opt}}}(n). \quad (14)$$

Assuming the input signal to be independent of the additive noise $t(n)$, the mean square error is given as [11]

$$Q(n) = Q_{\min} + \sum_{j=1}^P \sum_{l=1}^P E[\zeta_j(n)] \mathbf{R}_{j,l} E[\zeta_l(n)] + \sum_{j=1}^P \text{tr}\{\mathbf{R}_{j,l} E[\zeta_j(n) \zeta_j^T(n)]\}, \quad (15)$$

$\forall j = 1, 2, \dots, P \text{ and } j \neq l$

where

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