Int. J. Electron. Commun. (AEÜ) xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

International Journal of Electronics and Communications (AEÜ)

journal homepage: www.elsevier.com/locate/aeue

Regular Paper

Image deblurring and denoising by an improved variational model

Jianlou Xu [a,*], Aifen Feng [a], Yan Hao [a], Xuande Zhang [b], Yu Han [c]

[a] School of Mathematics and Statistics, Henan University of Science and Technology, Luoyang 471023, China
[b] School of Mathematics, Ningxia University, Yinchuan 750021, China
[c] College of Mathematic and Computational Science, Shenzhen University, Shenzhen 518060, China

ARTICLE INFO

Article history:
Received 19 June 2015
Accepted 16 May 2016
Available online xxxx

Keywords:
Total variation
Image denoising
Image deblurring
Staircase effect

ABSTRACT

Total variation method has been widely used in image processing. However, it produces undesirable staircase effect. To alleviate the staircase effect, some fourth order variational models were studied, which lead to the restored images smoothing and some details lost. In this paper, a low-order variational model for image deblurring and denoising is proposed, which is based on the splitting technique for the regularizer. Different from the general split technique, the improved variational model adopts the L_1 norm. To compute the new model effectively, we employ an alternating iterative method for recovering images from the blurry and noisy observations. The iterative algorithm is based on decoupling of deblurring and denoising steps in the restoration process. In the deblurring step, an efficient fast transforms can be employed. In the denoising step, the primal–dual method can be adopted. The numerical experiments show that the new model can obtain better results than those by some recent methods.

1. Introduction

The problem of image restoration has been widely studied in the last several years. The goal of image restoration is to recover the true image f from the observed noisy image

$$u_0 = Hf + \eta, \tag{1}$$

where u_0 is the observed noisy image, H is a bounded linear operator representing the convolution, and η denotes the additive Gaussian white noise.

Recovering f from u_0 is a typical example of an inverse problem. Since inverse problems are typically ill posed, a classical way to overcome ill-posed minimization problems is to add some regularization terms to the energy. This idea was firstly introduced by Tikhonov and Arsenin [1] as follows:

$$\min_f \int_\Omega |Hf - u_0|^2 dx + \frac{\lambda}{2} \int_\Omega |\nabla f|^2 dx, \tag{2}$$

where $\lambda > 0$ is a regularization parameter which balances the first and second terms. However, this model has very strong isotropic smoothing properties and tends to make images overly smooth, it often fails to adequately preserve important image attributes such as sharp edges. In order to overcome these drawbacks, the authors in [2] used the Total Variation (TV) of f instead of the L_2 norm of the gradient of f and proposed the following model

$$\min_f \int_\Omega |Hf - u_0|^2 dx + \lambda \int_\Omega |\nabla f| dx. \tag{3}$$

Although the TV regularizer has the ability of preserving the edges, it also gives rise to some undesired effects and transforms smooth signal into piecewise constant, the so-called staircase effects. In order to reduce the staircase effect, some high-order variational models were introduced [3–8], which contain the second order TV regularization terms. However, those high-order variational models need more complex boundary conditions.

Due to the nondifferentiability and nonlinearity of the TV function, Eq. (3) is more difficult to solve, some fast algorithms sprang up in recent years [9–13]. The authors in [9] used the variable-splitting and penalty techniques to solve the model. Ref. [10] and Ref. [11] put to use majorization–minimization method and alternating direction method for the TV image deblurring problems. In addition, the authors in [12,13] further studied the total bounded variational models for image deblurring and denoising problems. Nikolova et al. [14] studied nonconvex nonsmooth minimization methods for image restoration. There are also other methods for image deblurring, such as kernel regression [15], soft-thresholding method [16,17], nonlocal method [18], and wavelet method [19], etc.

Recently, to overcome the nondifferentiability and nonlinearity of the TV function of f in Eq. (3), Huang, Ng and Wen [20] intro-

* Corresponding author.
E-mail address: xujianlou@126.com (J. Xu).

http://dx.doi.org/10.1016/j.aeue.2016.05.008
1434-8411/

duced a new auxiliary variable u and proposed a fast TV minimization method as follows:

$$\min_{f,u} \frac{1}{2}\|Hf - u_0\|_2^2 + \frac{\lambda}{2}\|f - u\|_2^2 + \alpha \int_\Omega |\nabla u| dx, \tag{4}$$

where λ, α are positive regularization parameters. With this new auxiliary variable u, Eq. (4) can be solved effectively by decoupling of deblurring and denoising steps in the restoration process. In the deblurring step, fast transforms can be employed. In the denoising step, the TV model is solved by dual algorithm. Because the TV regularization term in Eq. (4) produces the staircase effect, in order to reduce it, the authors in [4] used the second order TV of u instead of the first order TV of u in Eq. (4) to design the following model

$$\min_{f,u} \frac{1}{2}\|Hf - u_0\|_2^2 + \frac{\lambda}{2}\|f - u\|_2^2 + \alpha \int_\Omega |\nabla^2 u| dx. \tag{5}$$

With the above model (5), the authors provided better results. However, the high-order TV regularizer causes some edges and details smoothed out, which are the very important characteristics in the restored images.

Inspired by the splitting idea [20], we introduce an auxiliary variable in the regularization term of Eq. (5) and divide the second order derivative term into two low order terms. The aim is that it not only can lower the order of image, but can alleviate the staircase effect. To solve the proposed model effectively, we also design an alternating iterative algorithm. From the experimental results, we see that the new model obtains better results than some current state-of-the-art methods. In addition, the new model's order is lower than the fourth order, so it does not need the more complex boundary condition than the fourth order diffusion equations.

In the rest of this paper, we will give the new model in Section 2. In Section 3, we do some numerical experiments to test our algorithm. Finally, Section 4 concludes this paper.

2. The proposed model and algorithm

2.1. The proposed model

From Eq. (4), we can see that it in fact splits the regularization term f of Eq. (3) into two terms by introducing an auxiliary variable u. When λ goes to infinity, the solution of Eq. (4) converges to that of Eq. (3). By the variable splitting, the operator of gradient and the operator of convolution can be computed respectively, and Eq. (4) can be solved by some fast algorithms effectively. Inspired by this idea, we introduce a new auxiliary variable $\boldsymbol{v}$ and propose the following model

$$\min_{u,\boldsymbol{v},f} \frac{\beta}{2} \int_\Omega (Hf - u_0)^2 dx + \frac{\lambda}{2}\|f - u\|_2^2 + \alpha_1 \int_\Omega |\nabla u - \boldsymbol{v}| dx + \alpha_2 \int_\Omega |\nabla \boldsymbol{v}| dx, \tag{6}$$

where β, λ, α_1, α_2 are the regularization parameters.

The proposed model has the following advantages: firstly, when $\alpha_1 \to \infty$, then $\boldsymbol{v} = \nabla u$, and Eq. (6) turns into Eq. (5), that is, it contains the second order TV, so it can reduce the staircase effect. When $\alpha_2 \to \infty$, then $\nabla \boldsymbol{v} \to 0$, the regularizer in Eq. (6) turns into the first order TV which is similar to Eq. (4), and it has the ability of preserving edges. All in all, Eq. (6) can automatically balance the first and second order terms by the parameters α_1, α_2, and it has the abilities of preserving the edges and reducing the staircase effect, which has been proved in [21,22].

Secondly, our variable splitting is different from Eq. (4) and [9]. We adopt the L_1 norm between vector $\boldsymbol{v}$ and the gradient of u not the L_2 norm. The advantage of this norm is that it can overcome the shortcoming of overly smooth, because the Euler–Lagrange of the L_2 norm produces the Laplace operator, which can smooth edges and details of the restored images.

From the above explanation, we can conclude that the proposed model provides a way of balancing between the first and second order of the objective function, so it can reduce the staircase effect while denoising. Meanwhile, it has the properties of edge preservation which is very important in image deblurring.

2.2. The proposed algorithm

To solve the proposed model (6), we use the following alternating direction method. The iterative algorithm is based on decoupling of denoising and deblurring steps in the image restoration process. It can be written into the following two minimization subproblems:

(1) Denoising step. For f fixed, find the solutions of $u, \boldsymbol{v}$

$$(u^{k+1}, \boldsymbol{v}^{k+1}) = \arg\min_{u,\boldsymbol{v}} \alpha_1 \int_\Omega |\nabla u - \boldsymbol{v}| dx + \alpha_2 \int_\Omega |\nabla \boldsymbol{v}| dx + \frac{\lambda}{2}\|f^k - u\|_2^2. \tag{7}$$

(2) Deblurring step. For u fixed, find the solution of f

$$f^{k+1} = \arg\min_f \frac{\beta}{2}\|Hf - u_0\|_2^2 + \frac{\lambda}{2}\|f - u^{k+1}\|_2^2. \tag{8}$$

We now give the corresponding algorithms for Eq. (7) and Eq. (8) respectively. First, for Eq. (7), by applying the Legendre–Fenchel transform, we obtain

$$\begin{aligned} &\arg\min_{u,\boldsymbol{v}} \alpha_1 \int_\Omega |\nabla u - \boldsymbol{v}| dx + \alpha_2 \int_\Omega |\nabla \boldsymbol{v}| dx + \frac{\lambda}{2}\|f^k - u\|_2^2 \\ &= \arg\min_{u,\boldsymbol{v}} \max_{\boldsymbol{p}\in P, \boldsymbol{q}\in Q} \langle \nabla u - \boldsymbol{v}, \quad \boldsymbol{p}\rangle + \langle \nabla \boldsymbol{v}, \quad \boldsymbol{q}\rangle + \frac{\lambda}{2}\|f^k - u\|_2^2, \end{aligned} \tag{9}$$

where $P = \{\boldsymbol{p} = (p_1, p_2)^T \,|\, |\boldsymbol{p}| \leqslant \alpha_1\}$, $\quad Q = \left\{\boldsymbol{q} = \begin{pmatrix} q_{11}, & q_{12} \\ q_{21}, & q_{22} \end{pmatrix} \,|\, |\boldsymbol{q}| \leqslant \alpha_2\right\}$, $\boldsymbol{p}$, $\boldsymbol{q}$ are the dual variables.

Applying the primal–dual method in [21,23] to Eq. (9), we can get the iterative schemes as follows:

$$\begin{cases} \boldsymbol{p}^{k+1} = proj_P(\boldsymbol{p}^k + \delta(\nabla \bar{u}^k - \bar{\boldsymbol{v}}^k)) \\ \boldsymbol{q}^{k+1} = proj_Q(\boldsymbol{q}^k + \delta(\nabla \bar{\boldsymbol{v}}^k)) \\ u^{k+1} = \frac{u^k + \tau\lambda f^k + \tau div \boldsymbol{p}^{k+1}}{1+\tau\lambda} \\ \boldsymbol{v}^{k+1} = \boldsymbol{v}^k + \tau(\boldsymbol{p}^k + div^h \boldsymbol{q}^{k+1}) \\ \bar{u}^{k+1} = 2u^{k+1} - u^k \\ \bar{\boldsymbol{v}}^{k+1} = 2\boldsymbol{v}^{k+1} - \boldsymbol{v}^k \end{cases} \tag{10}$$

where $proj_P(\tilde{\boldsymbol{p}}) = \frac{\tilde{\boldsymbol{p}}}{\max(1, |\tilde{\boldsymbol{p}}|/\alpha_1)}$, $\quad proj_Q(\tilde{\boldsymbol{q}}) = \frac{\tilde{\boldsymbol{q}}}{\max(1, |\tilde{\boldsymbol{q}}|/\alpha_2)}$ for any $\tilde{\boldsymbol{p}}, \tilde{\boldsymbol{q}}$; δ, τ are positive parameters.

Second, for Eq. (8), its corresponding Euler–Lagrange equation is

$$\beta H^T(Hf^{k+1} - u_0) + \lambda(f^{k+1} - u^{k+1}) = 0, \tag{11}$$

so we have

$$(\lambda I + \beta H^T H) f^{k+1} = (\beta H^T u_0 + \lambda u^{k+1}). \tag{12}$$

Because of the regularized term λI, the coefficient matrix $(\lambda I + \beta H^T H)$ is always invertible. We can obtain a closed solution for Eq. (12) as follows.

$$f^{k+1} = (\lambda + \beta H^T H)^{-1}(\beta H^T u_0 + \lambda u^{k+1}). \tag{13}$$

We note that when some boundary conditions, such as periodic boundary conditions, zero boundary conditions, et al., are applied

Please cite this article in press as: Xu J et al. Image deblurring and denoising by an improved variational model. Int J Electron Commun (AEÜ) (2016), http://dx.doi.org/10.1016/j.aeue.2016.05.008