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Optimizing power allocation in wireless networks: Are the implicit constraints really redundant?☆

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ABSTRACT

The widely considered power constraints on optimizing power allocation in wireless networks, e.g., non-negative individual power and limited sum of all the individual power, imply the constraints where each individual power is not greater than the limited sum. However, the related implicit constraints are generally regarded as redundant for algorithm design in most current studies. In this paper, we explore the question "Are the implicit constraints really redundant?" in the optimization of power allocation especially when using iterative methods (e.g., subgradient method) that have slow convergence speeds. Using the water-filling problem as an illustration, we first derive the structural properties of the optimal solutions based on Karush–Kuhn–Tucker conditions. Then we propose a non-iterative closed-form optimal method and use iterative methods (i.e., bisection method and subgradient method) to solve the problem. Our theoretical analysis shows that the implicit constraints are not redundant, and particularly, their consideration can effectively speed up the convergence of the subgradient method and reduce its sensitivity to the chosen step size. Numerical results for the water-filling problem and another existing power allocation problem demonstrate the effectiveness of considering the implicit constraints.

1. Introduction

Future wireless communication networks are required to support a large number of users with various requirements, especially with the growing demands of multimedia services [1–9]. To fulfill the requirements, radio resource management (RRM) plays an essential role as the system level control of co-channel interference and other radio transmission characteristics in wireless communication systems [10]. RRM involves strategies and algorithms for controlling parameters such as transmit power, user association, beamforming, data rate, handover criteria, modulation scheme and error coding scheme, etc., aiming at maximizing the utilization of the limited radio-frequency spectrum resources and radio network infrastructure [11]. Among these RRM techniques, power allocation optimization is a most important aspect of wireless communication system design and has been well studied in the past few decades.

On one hand, to solve various formulated power allocation problems or other optimization problems in wireless systems, the subgradient method is an iterative first-order method that has been widely used in many studies such as [12–26] and references therein. In [12,13], the subgradient method was used to solve the problem of maximizing the throughput under the constraints of interference power and individual transmit power in cognitive radio networks. In [14], subgradient methods were utilized based on dual decomposition to solve the simultaneous routing and resource allocation problem. In [15], a subgradient solution was developed to compute the maximum rate and the optimal routing strategy to solve the maximum multicast rate problem in the general undirected network model. In [16], a distributed subgradient method was used to solve the problem about how to choose opportunistic routes for users to optimize the total utility or profit of multiple simultaneous users in wireless mesh networks. In [17], distributed subgradient methods were applied to optimize global performance in delay tolerant networks with limited information. In [18], a subgradient solution was proposed to solve the problem of jointly optimizing channel pairing, channel-user assignment and power allocation in a single-relay multiple-access system. In [19], an α-approximation dual subgradient algorithm was proposed to optimize the total utility of multiple users in a load-constrained multihop wireless network. Based on the subgradient method,

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the study in [20] proposed a distributed optimal data gathering cost minimization framework with concurrent data uploading in wireless sensor networks. With the dual subgradient method, the study in [21] focused on convergence analysis of decentralized min-cost subgraph algorithms for multicasting in coded networks. In [22], the subgradient method was used to solve the joint power and bandwidth allocation problem in an improved amplify and forward cooperative communication scheme. In our previous work [23–26], the subgradient method was also used to design resource allocation or scheduling schemes with different optimization objectives under the network constraints in small cell systems. Though subgradient methods can be operated in a distributed manner, they usually have slow convergence rates and are very sensitive to the chosen iteration step sizes [27,28], which need to be improved to reduce the computation costs and even signaling overhead in wireless networks and to reduce the sensitivity to the chosen step sizes since (1) improper step sizes may not make the subgradient methods converge and (2) proper step sizes are not easy to choose especially when the formulated optimization problems are very complex.

On the other hand, mathematically, the formulated optimization problems of power allocation in wireless systems are generally subject to at least two inequality constraints [10,12–24] on p_n, the transmit power allocated at a base station (BS) for the nth user, e.g., (1) nonnegative: $p_n \geq 0, \forall n$, and (2) limited sum: $\sum_{n=1}^{N} p_n \leq P_{\max}$, where N and $P_{\max}$ respectively denote the total number of users served by the BS and the BS's maximum transmit power. These two power constraints imply another set of (implicit) constraints, i.e., $p_n \leq P_{\max}, \forall n$. However, in most currently studied power allocation optimization problems or other similar optimization problems with the above two inequality constraints, the implicit constraints are regarded as redundant and useless in the design of strategies and algorithms for solving the problems. From the perspective of mathematics, the implicit constraints obviously hold, but are they really redundant in optimization algorithms? To the best of our knowledge, this question is unexplored.

The above motivates us to answer the question "Are the implicit constraints really redundant?" in power allocation optimization especially when using subgradient methods in the solution algorithms. Specifically, we study the water-filling problem as a typical illustration of power allocation optimization. Based on Karush–Kuhn–Tucker (KKT) conditions, we derive the structural properties of the optimal solutions to the water-filling problem and evaluate the performance of the proposed methods with/without the consideration of the implicit constraints. As the extension of our previous work [29], our contributions of this paper are summarized below:

- This paper is the first to explore the question "Are the implicit constraints really redundant?" in power allocation optimization especially when using subgradient methods.
- By illustrating the water-filling problem typical in resource allocation, our theoretical analysis shows that considering the implicit constraints can effectively speed up the convergence of the subgradient methods, reduce the sensitivity to the chosen step size and lead to convergence even when an improper step size is used, while the opposite is true if the implicit constraints are not considered. This finding can be extended to other optimization problems and applied to other iterative methods. Besides, we propose a non-iterative closed-form optimal method and iterative bisection methods.
- Numerical results on the water-filling problem and the power allocation problem for multiuser systems in [30] show that considering the implicit constraints in the algorithm design can effectively improve the performance of the used subgradient methods.

The rest of this paper is organized as follows. In Section 2, we formulate the water-filling problem as an illustration of power allocation. In Section 3, we derive the structural properties of the optimal solutions. In Section 4, algorithms for solving the optimization problem are proposed and analyzed. All the possible cases are theoretically analyzed with some well-designed examples in Section 5. Experiment results are shown to evaluate the performance of the proposed algorithms in Section 6. Finally, Section 7 gives the conclusions.

2. The water-filling problem typical in resource allocation

In this section, we provide a general form of resource allocation problems and illustrate a typical resource allocation problem, i.e., water-filling problem, to explore whether the implicit constraints are really redundant for algorithm design.

2.1. General resource allocation problem

Many existing power allocation or other resource allocation optimization problems can be formulated or transformed in a general form as

$$\max_{\mathbf{p},\mathbf{y}} f(\mathbf{p},\mathbf{y}) \tag{1a}$$

$$s.t. \quad p_n \geq 0, \quad \forall n \in \mathcal{N}, \tag{1b}$$

$$\sum_{n=1}^{N} p_n \leq P_{\max}, \tag{1c}$$

$$g_i(\mathbf{p},\mathbf{y}) \leq 0, \quad \forall i \in \mathcal{I}, \tag{1d}$$

$$\mathbf{y} \in \mathcal{S}_Y, \tag{1e}$$

where N is a given number (e.g., number of users), $\mathcal{N} = \{1, 2, \ldots, N\}$, $\mathcal{I}$ and $\mathcal{S}_Y$ are two given sets about resource constraints; $\mathbf{p} = [p_1, p_2, \ldots, p_N]^T$ and $\mathbf{y}$, respectively, are variable vectors of power and other resource allocations; $f(\mathbf{p},\mathbf{y})$ and $g_i(\mathbf{p},\mathbf{y})$ are, respectively, the given objective function (e.g., sum data rate) and constraint functions w.r.t. $\mathbf{p}$ and $\mathbf{y}$; $P_{\max}$ is a positive constant scalar (e.g., maximum transmit power).

From (1b) and (1c), we can get the implicit constraints as

$$p_n \leq P_{\max}, \quad \forall n \in \mathcal{N}. \tag{2}$$

Remark 1. In existing studies, the same or similar implicit constraints in (2) are usually overlooked and are regarded as redundant.

Whether the problems in (1) are convex or nonconvex, they can be solved with a family of iterative methods (e.g., subgradient methods) to get the optimal or suboptimal solutions.

2.2. Water-filling problem

To explore whether the implicit constraints are redundant, we illustrate a most typical resource allocation optimization problem, i.e., the water-filling problem, which is to maximize the sum of the capacity of users under transmit power constraints [31] and can be formulated as

$$\max_{\mathbf{p}} \sum_{n=1}^{N} \log_2(1+\alpha_n p_n) \tag{3a}$$

$$s.t. \quad p_n \geq 0, \quad \forall n \in \mathcal{N}, \tag{3b}$$

$$\sum_{n=1}^{N} p_n \leq P_{\max}, \tag{3c}$$