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# Critical journey evolving graphs

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ABSTRACT

End-to-end connectivity in dynamic networks may not hold at a given point of time. Nevertheless, temporal paths (or journeys) may still exist over time and space due to the inherent evolution of networks. Large amount of work has been done to examine foremost journeys and connectivity of dynamic networks. However, as far as we know, there is still no work which can address foremost journeys calculation in continuous time under scenarios where contacts have arbitrary durations and edge traversal time is non-negligible. To this end, we propose a model named *critical journey evolving graphs* (CJEGs) to effectively characterize the temporal connectivity of dynamic networks. By CJEGs, a foremost journey between any pair of nodes starting at any time can be inferred directly. Accordingly, temporal connectivity metrics such as velocity, efficiency and density can be figured out. To construct CJEGs, a distributed algorithm called CJEG-PERST is developed, which can update CJEGs online adaptively with the evolution of networks, thus avoiding the hassle of heavy computation and memory overhead. We conduct extensive experiments and analysis to explore the temporal connectivity of dynamic networks by applying CJEGs to synthetic and realistic datasets, as well as making comparison with other models such as *temporal reachability graphs* (TRGs). Experimental results show CJEGs yield many fresh and enlightening insights into connectivity of dynamic networks. Especially, two important applications of CJEGs are further discussed, namely how to derive TRGs and calculate theoretical performance benchmark for opportunistic routing. These explorations preliminarily demonstrate the promising potentials of the CJEGs model.



## 1. Introduction

In the past few years, highly dynamic networks, and especially opportunistic networks [1], are becoming an important research branch of complex systems. A key feature of such networks is that their topologies evolve over time. End-to-end connectivity may not hold at a given point of time, but temporal paths, also formally named journeys [2], may still exist over time and space, thus communication between any two nodes is still likely to occur by store-carry-forward paradigm [3].

To explore time evolution behaviors of dynamic networks, time-varying graphs (TVGs) [2], alternatively called evolving graphs [4], or temporal networks [5,6] have emerged to serve as an elementary model for such networks. Fig. 1(a) shows an example of TVGs $\mathcal{G}_1$ which represents a three-node dynamic network. Labels on edges denote time intervals during which contacts occur between two nodes. An example of a journey denoted as $\{(ba, 1), (ac, 3)\}$ (red dotted lines) represents a temporal path between $(b, c)$ which departs from node $b$ for node $a$ at time $t = 1$ s, arrives at node $a$ at $t = 2$ s and lingers for some time, then leaves node $a$ for node $c$ at $t = 3$ s and reaches destination $c$ at $t = 4$ s (Unless otherwise specified, we assume that one-hop transmission delay $\tau$ is 1 s in this paper).

Based on TVGs, intensive research efforts have been made for dynamic networks. Among these studies, identification of influential nodes and exploration of time-evolving connectivity attract more attentions. Kim et al. [7] observe the temporal node centrality using time-ordered graphs. Magnien et al. [8] explore the evolution of nodes centrality in dynamic networks. Some works [9,10] explore the connectivity of dynamic networks in terms of temporal reachability. Whitbeck et al. [10] propose a model named temporal reachability graphs (TRGs), from which the reachability between nodes can be observed in continuous time, hence allowing the exploration of time-evolving connectivity. However, TRGs just offer a binary answer with respect to reachability, without providing further detailed information such as journey hops and

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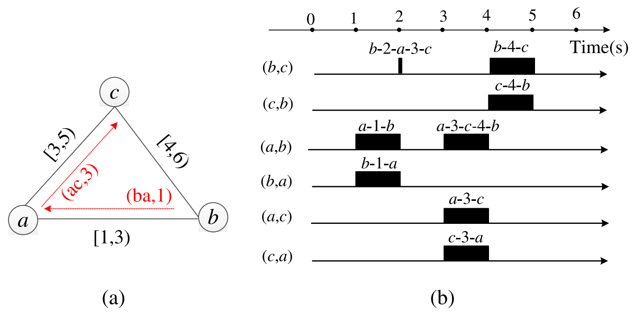


**Fig. 1.** A TVG and its corresponding CJEG. (a) TVG $\mathcal{G}_1$. (b) $\mathcal{C}_\tau$ derived from $\mathcal{G}_1$ ($\tau$ = 1 s).

reachability time. The approach to build TRGs in [10] is an approximate algorithm which is not capable of keeping journey information.

Due to the fundamental significance of foremost journey (formal definition is provided in Section 3.1) in temporal connectivity exploration, many previous works focus on finding foremost journeys at a given point of time, under the assumption that a full knowledge of network dynamics is known in advance [4,11,12]. For example, Xuan et al. propose an approach to compute foremost journeys in [4] and Jain et al. develop an algorithm to compute shortest (minimum cost) paths in [11]. These works address the issue of foremost journeys calculation from one source to all sinks at a fixed time.

In dynamic networks, however, foremost journeys between any pair of nodes are time dependent. With time evolving, foremost journeys between given pair of nodes may be different. Take $(b, c)$ in $\mathcal{G}_1$ as an example, at time $t = 1$ s, a foremost journey from $b$ to $c$ is $\{(ba, 1), (ac, 3)\}$; while at time $t = 4$ s, a foremost journey from $b$ to $c$ is $\{(bc, 4)\}$. To explore the variation of foremost journeys and temporal reachability between nodes, it is necessary to develop more efficient approaches to calculate foremost journeys at any given point of time.

To this end, one may reduce a dynamic network to a series of static snapshots [7,12]. For such snapshots, existing algorithms for static graphs can be extended to calculate foremost journeys. However, the reduction process enables these static graphs models to be just a heuristic way for discrete times. This is not sufficient for the exploration of continuous time-evolving connectivity of dynamic network. In this paper, we are mainly concerned with solving such problem of "foremost journeys calculation in continuous time". It is worth noting that the problem is observed in a universal scenario, where time is continuous, contacts have arbitrary durations and edge traversal time (i.e. one-hop transmission delay $\tau$) is non-negligible.

Solving the problem of "foremost journeys calculation in continuous time" is of great significance, which could provide effective approaches to address some important issues in dynamic networks. Once the foremost journey at any point of time is derived, temporal distance between any pair of nodes at any time can be calculated out easily. Thus a number of connectivity metrics including temporal reachability [10], temporal efficiency [8], average density [10], etc., which are difficult to obtain in a universal scenario previously, can be inferred conveniently. Novel temporal centrality metrics [7] can also be designed to identity the important nodes of temporal networks. These would be highly beneficial to off-line theoretic analysis of dynamic networks, and on-line transmission schemes designing as well. For example, performance benchmark [13] for routing protocols in dynamic networks can be obtained from a theoretical perspective. An opportunistic network node may maintain the foremost journeys in continuous time, upon which temporal distance between nodes can be predicted online, thus the optimal forwarding and caching decisions can be made.

Calculating foremost journeys at any time, however, is nontrivial. To the best of our knowledge, there is no algorithm existed currently which can build all foremost journeys for a given dynamic network in a universal scenario. After deliberate analysis, we discover that foremost journeys in continuous time can be obtained once some critical foremost journeys (or simply critical journeys) are figured out. Take $(b, c)$ in $\mathcal{G}_1$ for example. At time $t = 0$ s, all journeys $\{(ba, t), (ac, 3)\}$ $(t \in [1, 2])$ are foremost journeys between $(b, c)$. Among these journeys, $(ba, 2), (ac, 3)$ is a journey whose departure time from node $b$ is the latest. So long as we figure out this journey, the foremost journey (i.e. $\{(ba, 2), (ac, 3)\}$) between $(b, c)$ at any time $t \in [0, 2]$ can be identified, which will dramatically reduce the overhead for foremost journeys calculation.

In this paper, we propose a model named critical journey evolving graphs (CJEGs), in which only critical journeys are kept, to characterize network dynamics and connectivity. Formally, given a TVG $\mathcal{G}$, a critical journey $\mathcal{J}_{u,v}$ exists from $u$ to $v$ at time $t$ in its derived CJEG $\mathcal{C}_\tau$ ($\tau$ is the edge traversal time) if $\mathcal{J}_{u,v}$ is a foremost journey at time $t$, also, among all foremost journeys with the same arrival time $\mathcal{J}_{u,v}$ has the latest departure time (formal definition is provided in Section 3.2).

An example of CJEGs $\mathcal{C}_\tau$ derived from Fig. 1(a) is illustrated in Fig. 1(b), where $\tau$ is the edge traversal time. The vertical line on $(b, c)$ labeled with "b-2-a-3-c" denotes that a critical journey $\{(ba, 2), (ac, 3)\}$ exists at time $t = 2$ s, while the rectangle on $(a, b)$ labeled with "a-3-c-4-b" denotes a series of critical journeys exist during the time interval [3,4]. Among all these journeys, any journey can be seen as a shifting journey of $\{(ac, 3), (cb, 4)\}$ (such shifting journeys, e.g. $\{(ac, 3.5), (cb, 4.5)\}$, pass the same nodes as $\{(ac, 3), (cb, 4)\}$, but with a shifting time leaving the corresponding nodes).

Critical journeys are a special kind of foremost journeys that cannot be put off. If a message leaves source later than a critical journey $\mathcal{J}_{u,v}$, it definitely arrives at the destination later than $\mathcal{J}_{u,v}$. If all critical journeys are sorted on the timeline according to the departure time from the source, then all these journeys remain orderly according to the arrival time to the destination. This property guarantees that a foremost journey from $u$ to $v$ at time $t$ is exactly the first critical journey after time $t$ ($t$ is inclusive) on the timeline. Take node pair $(b, c)$ in Fig. 1(b) for example: when $t \in [0, 2]$, a foremost journey between $(b, c)$ at time $t$ is $\{(ba, 2), (ac, 3)\}$; when $t \in [2, 4]$, a foremost journey between $(b, c)$ at time $t$ is $\{(bc, 4)\}$; while when $t \in [4, 5]$, a foremost journey between $(b, c)$ at time t is $\{(bc, t)\}$. Consequently, once the CJEG of $\mathcal{G}$ is derived, a foremost journey between any pair of nodes at any given time can be inferred directly. As such, CJEGs provide an intuitive view of foremost journeys in continuous time.

The main contributions are as follows:

- To investigate connectivity of dynamic networks in continuous time, a theoretic model named CJEGs is proposed. We analyze the properties of CJEGs in detail. Particularly, the condition under which critical journeys exist between two nodes is explored. Such condition lays an important foundation for the calculation of CJEGs.
- To efficiently construct CJEGs for a common dynamic network where contacts have durations and edge traversal time is non-negligible, a distributed algorithm named CJEG-PERST is developed. The algorithm can adaptively and progressively build CJEGs online with the evolution of networks, thus avoiding the hassle of significant computation and memory overhead. Moreover, based on CJEG-PERST, we develop a tool named CjegSim, which can be used to calculate CJEGs offline if only a