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A two stage fuzzy piecewise logistic model for penetration forecasting

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ABSTRACT

It is undeniably crucial for a firm to be able to make a forecast regarding the sales volume of new products. However, the current economic environments invariably have uncertain factors and rapid fluctuations where decision makers must draw conclusions from minimal data. Previous studies combine scenario analysis and technology substitution models to forecast the market share of multigenerational technologies. However, a technology substitution model based on a logistic curve will not always fit the *S* curve well. Therefore, based on historical data and the data forecast by both the Scenario and Delphi methods, a two stage fuzzy piecewise logistic growth model with multiple objective programming is proposed herein. The piecewise concept is adopted in order to reflect the market impact of a new product such that it can be possible to determine the effective length of sales forecasting intervals even when handling a large variation in data or small size data. In order to demonstrate the model's performance, two cases in the Television and Telecommunication industries are treated using the proposed method and the technology substitution model or the Norton and Bass diffusion model. A comparison of the results shows that the proposed model outperforms the technology substitution model and the Norton and Bass diffusion model.

1. Introduction

The replacement of an older generation of technological products with newer ones is common in the high-technology sector. To better understand and describe this phenomenon, Fisher and Pry [1] proposed the technological substitution model in 1971 which analyzes the penetration process of new-generation technologies replacing old ones. Marchetti and Nakicenovic [2] expanded upon this model [1] to deal with multi-generation and multi-product conditions. A more comprehensive description of the model and its assumptions has been given by Nakicenovic [3]. Notably, these substitution models are market share models which assume that there is a market there to be substituted. From the innovation diffusion perspective, Norton and Bass [4] developed a multi-generation diffusion model which has the ability to estimate or forecast market potential and has become a popular forecasting method in the field.

A key proposal of Marchetti and Nakicenovic [2] was that the life cycle of every technology includes three phases: growth, saturation and decline. During the growth phase of a new technology, substitution proceeds logistically. Finally, the technology enters the decline phase and degenerates logistically. However, in the current competitive environment, it is hard for the product life cycle to be represented by a smooth *S* curve or fitted logistic curve [5,6]. Tseng et al. [5], proposed three scenarios, the optimistic, pessimistic and most possible scenarios, for the development of organic light-emitting diode (OLED), a television (TV) technology with no sales data. Then, they applied the Marchetti and Nakicenovic [2] technological substitution model, in which the diffusion curves of all technologies are supposed to be logistic, to forecast the market share of the color cathode ray tube (CRT), rear projection (RP), plasma display panel (PDP), liquid crystal display (LCD) and OLED TVs. They found that the diffusion curve of the PDP TV was not a logistic curve, but an *S* curve, which is unable to meet the assumptions of Marchetti and Nakicenovic's technological substitution model [2] and the forecasting performance was not very good (Please refer to the MAPE of PDP TV in Table 2). Therefore, one of the research gaps exists insofar as the conventional technological substitution model was hard to deploy logistically.

In addition to the diffusion curve issue as regards new products, the other research gap was such that the analysis of uncertainty in the environment also requires consideration [7,8]. To handle the uncertainty issue, two different perspectives are addressed. Tanaka et al. [9] developed fuzzy regression with symmetrical triangular fuzzy parameters *via* a possibility distribution to provide

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possibility forecasting intervals to decision makers. However, the conventional fuzzy regression model is sensitive to outliers, *i.e.* the more the various data fluctuate, the wider the forecast intervals, a situation which then causes difficulties with regards to interpretation [10]. In order to cope with large variations in data, Yu et al. [11] proposed the fuzzy piecewise regression model which behaves differently in different parts of the range of crisp input variables. Huang and Tzeng [12] applied this model to their two-stage fuzzy piecewise regression analysis to better grasp the dynamics of a nonlinear time-series within a fuzzy environment. Their objective was to forecast product life time (PLT) and the annual shipment of products during the entire PLT of multiple generation products. However, the number of change-points could not be generated automatically. So, Yu and Tzeng [13] extended the fuzzy piecewise regression model to overcome this limitation. Based on the current research gap, we propose a fuzzy piecewise logistic growth model which combines the advantages of a piecewise regression model and the logistic diffusion model so as to deal with complex growth for single- or multiple-generation(s) product sales forecasting [14]. This model can solve the problem that arises when the time series shows an *S* curve but not a logistic curve and can be applied to multiple-generation diffusion forecasting.

Scenario analysis, another perspective to deal with uncertainty, is an important forecasting method used since the 1960s to forecast and understand future developments. The approach outlines some aspects of the future world by telling stories to emphasize dimensions of today's world under uncertainties [15]. Thus, Scenario analysis presents rich and complex portraits of possible future scenarios; however, it fails to provide quantifiable forecasts [16]. Therefore, Wang and Lan [17] combined Scenario analysis (which addresses an uncertain future) and the technological substitution model (a quantitative method) to analyze the development of new-generation technologies. The limitation of their approach, which pertains to Scenario analysis, is such that scenarios are commonly constructed on the basis of expert opinion and often there are large differences among these opinions.

To deal with this diversity, the Delphi technique was developed to obtain the most reliable consensus within a group of experts by using a series of intensive questionnaires interspersed with controlled opinion feedback [18]. Subsequently, some research has combined Scenario analysis and Delphi method to generate future scenarios [19–24]. However, these scenarios are still unable to provide clear quantified information such as the forecast market share of the new technology. Therefore, Tseng et al. [5] combined Scenario analysis with the technological substitution model and the Delphi method, which is a kind of interpolation technique used to analyze the development of a new technology, namely, the latest-generation TV, or the OLED TV. Additionally, they request that the experts forecast the market share to solve the problem of limited data as regards new technology. However, their technology substitution model based on a logistic curve will not always fit the *S* curve well.

As such, there are two research gaps: the technological substitution model needs to be a logistic curve and the newest technology has limited data; the future brings uncertainty and sales forecasting need to consider more scenarios. Therefore, this work aims to integrate the advantages of Scenario analysis and the Delphi method with the new proposed fuzzy piecewise logistic model to analyze the development of the latest-generation technology. In order to demonstrate the satisfactory performance of the proposed model, we applied it to Tseng et al.'s two cases [5,25] and adopted the data from the Scenario analysis and the Delphi method in Tseng et al.'s work. In the first case, the proposed method forecasts the futures of CRT, RP, PDP, LCD and OLED TVs over the next ten years. In the second case, the market shares of second generation (2G) mobile, third generation (3G) mobile, Microwave Access (WiMAX) and long-term evolution (LTE) over the next ten years are forecast.

The structure of the rest of the paper is as follows. Section 2 presents a fuzzy piecewise logistic regression model. Section 3 describes the proposed methodology. Section 4 presents two empirical cases. Section 5 concludes.

2. Fuzzy piecewise logistic regression model and the forecasting efficiency measures

The fuzzy piecewise logistic growth model obtains the smallest interval forecast which includes all of the raw data to compensate for the weakness of the Norton and Bass model [4]. Therefore, it can provide a possible interval and is a viable alternative to the forecast for multiple-generations. Moreover, it can detect the change points in a complex growth process.

2.1. Fuzzy piecewise logistic regression model [14]

Based on the fuzzy piecewise regression model [11], an initial fuzzy piecewise logistic regression model is described as follows:

$$\hat{\pi}_t = 1/(1+\exp(-A_1 t + A_0 + \sum_{s=1}^{n-1} B_s(|t-p_s| + t - p_s)/2)) \tag{1}$$

where $\hat{\pi}_t$ is the forecast market share; A_0, A_1 and B_s for $s = 1, \ldots, n-1$ are fuzzy coefficients; h is cut level; and $\hat{\pi}_t$ and π_t are forecast fuzzy output and raw data, respectively, *i.e.*, $\pi_t \subseteq \hat{\pi}_t$. The upper bounds and lower bounds of the piecewise logistic diffusion model are presented below, respectively:

$$\begin{aligned}\hat{\pi}_t^U &= 1/(1+\exp(-a_{1c}t + a_{0c} + \sum_{s=1}^{n-1} b_{sc}(|t-p_s| + t - p_s)/2 \\ &\quad -(1-h)(a_{1w}t + a_{0w} + \sum_{i=1}^{n-1} b_{sw}(|t-p_s| + t - p_s)/2))\end{aligned} \tag{2}$$

$$\begin{aligned}\hat{\pi}_t^L &= 1/(1+\exp(-a_{1c}t + a_{0c} + \sum_{s=1}^{n-1} b_{sc}(|t-p_s| + t - p_s)/2 \\ &\quad +(1-h)(a_{1w}t + a_{0w} + \sum_{s=1}^{n-1} b_{sw}(|t-p_s| + t - p_s)/2))\end{aligned} \tag{3}$$

The inclusion relationships among the upper and lower bounds of the piecewise logistic diffusion model and the raw data are as follows:

$$\hat{\pi}_t^L \subseteq \pi_t \subseteq \hat{\pi}_t^U \tag{4}$$

After taking logarithms, the fuzzy piecewise logistic growth model is transformed into a piecewise linear model which meets the requirements of the inclusion relationship, according to which the fuzzy regression must include the observations, *i.e.*, $\hat{Y}_t^L \subseteq Y_t \subseteq \hat{Y}_t^U$:

$$\begin{aligned}\hat{Y}_t^L &= \ln\left(\frac{1-\hat{\pi}_t^U}{\hat{\pi}_t^U}\right) = -a_{1c}t + a_{0c} + \sum_{s=1}^{n-1} b_{sc}(|t-p_s| + t - p_s)/2 \\ &\quad -(1-h)(a_{1w}t + a_{0w} + \sum_{i=1}^{n-1} b_{sw}(|t-p_s| + t - p_s)/2)\end{aligned} \tag{5}$$