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Linear programming approach for solving fuzzy critical path problems with fuzzy parameters

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ABSTRACT

To the best of our knowledge, there is no method in the literature to find the fuzzy optimal solution of fully fuzzy critical path (FFCP) problems i.e., critical path problems in which all the parameters are represented by *LR* flat fuzzy numbers. In this paper, a new method is proposed for the same. Also, it is shown that it is better to use *JMD* representation of *LR* flat fuzzy numbers in the proposed method as compared to the other representation of *LR* flat fuzzy numbers.

1. Introduction

In today's highly competitive business environment, project management's ability to schedule activities and monitor progress within strict cost, time and performance guidelines is becoming increasingly important to obtain competitive priorities such as on-time delivery and customization. In many situations, projects can be complicated and challenging to manage. When the activity times in the project are deterministic and known, critical path method (CPM) has been demonstrated to be a useful tool in managing projects in an efficient manner to meet this challenge. The purpose of CPM is to identify critical activities on the critical path so that resources may be concentrated on these activities in order to reduce the project length time.

The successful implementation of CPM requires the availability of a clear determined time duration for each activity. However, in practical situations this requirement is usually hard to fulfill, since many of activities will be executed for the first time. To deal with such real life situations, Zadeh [25] introduced the concept of fuzzy set. Since there is always uncertainty about the time duration of activities in the network planning, due to which fuzzy critical path method (FCPM) was proposed since the late 1970s.

For finding the fuzzy critical path, several approaches are proposed over the past years. The first method called FPERT, was proposed by Chanas and Kamburowski [1]. They presented the project completion time in the form of fuzzy set in the time space. Gazdik [7] developed a fuzzy network of unknown project to estimate the activity durations and used fuzzy algebraic operators to calculate the duration of the project and its critical path. Kaufmann and Gupta [9] devoted a chapter of their book to the critical path method in which activity times are represented by triangular fuzzy numbers. McCahon and Lee [20] presented a new methodology to calculate the fuzzy completion project time.

Nasution [21] proposed how to compute total floats and find critical paths in a project network. Yao and Lin [24] proposed a method for ranking fuzzy numbers without the need for any assumptions and have used both positive and negative values to define ordering which then is applied to CPM. Dubois et al. [5] extended the fuzzy arithmetic operational model to compute the latest starting time of each activity in a project network. Lin and Yao [17] introduced a fuzzy CPM based on statistical confidence-interval estimates and a signed distance ranking for $(1-\alpha)$fuzzy number levels. Liu [19] developed solution procedures for the critical path and the project crashing problems with fuzzy activity times in project planning. Liang and Han [16] presented an algorithm to perform fuzzy critical path analysis for project network problem. Zielinski [26] extended some results for interval numbers to the fuzzy case for determining the possibility distributions describing latest starting time for activities. Chen [2] proposed an approach based on the extension principle and linear programming (LP)

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formulation to critical path analysis in networks with fuzzy activity durations. Chen and Hsueh [3] presented a simple approach to solve the CPM problems with fuzzy activity times (being fuzzy numbers) on the basis of the linear programming formulation and the fuzzy number ranking method that are more realistic than crisp ones. Yakhchali and Ghodsypour [23] introduced the problems of determining possible values of earliest and latest starting times of an activity in networks with minimal time lags and imprecise durations that are represented by means of interval or fuzzy numbers.

In this paper, a new method is proposed to find the fuzzy optimal solution of fully fuzzy critical path (FFCP) problems i.e., critical path problems in which all the parameters are represented by *LR* flat fuzzy numbers. Also, it is shown that it is better to use *JMD* representation of *LR* flat fuzzy numbers in the proposed method as compared to the other representation of *LR* flat fuzzy numbers.

This paper is organized as follows: In Section 2, some basic definitions and arithmetic operations are presented. In Section 3, linear programming formulation of crisp critical path (CCP) problems and fuzzy critical path (FCP) problems are presented. The linear programming formulation of FFCP problems are proposed in Section 4. In Section 5, a new method is proposed to find the fuzzy optimal solution of FFCP problems and also the validity and advantages of the proposed method is discussed. In Section 6, *JMD* representation of *LR* flat fuzzy numbers is presented and advantages of *JMD* representation of *LR* flat fuzzy numbers over other existing representation of *LR* flat fuzzy numbers are discussed. Conclusion is discussed in Section 7.

2. Preliminaries

In the literature [6,27], it is pointed out that the computational efforts required to solve a fuzzy linear programming problem can be reduced, if the decision maker express his data using *LR* flat fuzzy numbers. All kinds of crisp numbers, triangular and trapezoidal fuzzy numbers are *LR* flat fuzzy numbers. So, *LR* flat fuzzy numbers are frequently used to increase the computational efficiency without limiting the generality beyond the acceptable limits and facilities the ease of acquisition of data to solve real life problems.

In this section, some basic definitions and arithmetic operations between two *LR* flat fuzzy numbers are presented.

2.1. Basic definitions

In this section, some basic definitions are presented.

Definition 1. [8] Let X be a classical set of objects. Then, the set of ordered pairs $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : x \in X\}$, where $\mu_{\widetilde{A}} : X \to [0, 1]$, is called a fuzzy set in X. The evaluation function $\mu_{\widetilde{A}}(x)$ is called the membership function.

Definition 2. [8] Let $\widetilde{A}$ be a fuzzy set in X and $\lambda \in [0, 1]$ be a real number. Then, a classical set $A^{\lambda} = \{x \in X : \mu_{\widetilde{A}}(x) \geq \lambda\}$ is called a λ-level set or λ-cut of $\widetilde{A}$.

Definition 3. [8] A fuzzy set $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : x \in X\}$ is called a normalized fuzzy set if and only if $\underset{x \in X}{\text{Supremum}}\{\mu_{\widetilde{A}}(x)\} = 1$.

Definition 4. [8] A fuzzy set $\widetilde{A}$ is called a convex fuzzy set if and only if $\mu_{\widetilde{A}}(\alpha x_1 + (1-\alpha)x_2) \geq \text{minimum}\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}\ \forall x_1, x_2 \in X,\ \alpha \in [0, 1]$.

Definition 5. [6] A convex normalized fuzzy set $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : x \in \mathbb{R}\}$ on the real line $\mathbb{R}$ is called a fuzzy number if and only if $\mu_{\widetilde{A}}(x)$ is piecewise continuous in $\mathbb{R}$.

Definition 6. [8] A fuzzy number $\widetilde{A}$ is said to be a non-negative fuzzy number if and only if $\mu_{\widetilde{A}}(x) = 0\ \forall x < 0$.

Definition 7. [6] A function $L : [0, \infty) \to [0, 1]$ (or $R : [0, \infty) \to [0, 1]$) is said to be reference function of fuzzy number if and only if

(i) $L(0) = 1$ (or $R(0) = 1$)
(ii) L (or R) is non-increasing on $[0, \infty)$.

Definition 8. [6] A fuzzy number $\widetilde{A}$ defined on universal set of real numbers $\mathbb{R}$, denoted as $(\underline{a}, \overline{a}, a^L, a^R)_{LR}$, is said to be an *LR* flat fuzzy number if its membership function $\mu_{\widetilde{A}}(x)$ is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} L(\dfrac{\underline{a} - x}{a^L}), & x \leq \underline{a},\ a^L > 0 \\ R(\dfrac{x - \overline{a}}{a^R}), & x \geq \overline{a},\ a^R > 0 \\ 1, & \underline{a} \leq x \leq \overline{a} \end{cases}$$

Definition 9. [6] Let $\widetilde{A} = (\underline{a}, \overline{a}, a^L, a^R)_{LR}$ be an *LR* flat fuzzy number and α be a real number in the interval [0, 1]. Then, the crisp set $A_{\alpha} = \{x \in X : \mu_{\widetilde{A}}(x) \geq \alpha\} = [\underline{a} - a^L L^{-1}(\alpha), \overline{a} + a^R R^{-1}(\alpha)]$, is said to be an α-cut of $\widetilde{A}$.

Definition 10. [6] An *LR* flat fuzzy number $\widetilde{A} = (\underline{a}, \overline{a}, a^L, a^R)_{LR}$ is said to be a zero *LR* flat fuzzy number if and only if $\underline{a} = 0, \overline{a} = 0, a^L = 0$ and $a^R = 0$.

Definition 11. [6] Two *LR* flat fuzzy numbers $\widetilde{A_1} = (\underline{a_1}, \overline{a_1}, a_1^L, a_1^R)_{LR}$ and $\widetilde{A_2} = (\underline{a_2}, \overline{a_2}, a_2^L, a_2^R)_{LR}$ are said to be equal i.e., $\widetilde{A_1} = \widetilde{A_2}$ if and only if $\underline{a_1} = \underline{a_2}, \overline{a_1} = \overline{a_2}, a_1^L = a_2^L$ and $a_1^R = a_2^R$.