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Minimum cost dominating tree sensor networks under probabilistic constraints

Pablo Adasme [a,*], Rafael Andrade [b], Abdel Lisser [c]

[a] Departamento de Ingeniería Eléctrica, Universidad de Santiago de Chile, Avenida Ecuador, 3519 Santiago, Chile
[b] Departamento de Estatística e Matemática Aplicada, Universidade Federal do Ceará, Campus do Pici, BL 910, CEP 60.455-760 Fortaleza, Ceará, Brazil
[c] Laboratoire de Recherche en Informatique, Université de Paris-Sud 11, Ada Lovelace, Paris 91405, France

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ABSTRACT

There is an increasing interest in planning sensor networks by considering both the impact of distances among sensors and the risk that power consumption leads to a very small network lifetime. A sensor failure can affect sensors in its neighborhood and compromise the network data communication. Weather conditions may cause the power consumption of data communication to vary with uncertainty. This work introduces a compact probabilistic optimization approach to handle this problem while considering jointly or separately dependence among power consumption of the links of the network in a unified framework. We explore the concept of copulas in a dominating arborescence (DA) model for directed graphs, extended accordingly to handle the uncertain parameters. We give a proof of the DA model correctness and show that it can solve to optimality some benchmark instances of the deterministic dominating tree problem. Numerical results for the probabilistic approach show that our model tackles randomly generated instances with up to 120 nodes.

1. Introduction

A classic version of the dominating tree (DT) sensor network problem [24] concerns determining a minimum cost tree T in an undirected graph $G = (V, E)$ of set of nodes V and set of edges E, with $c_e \geq 0$ being the cost (weight–distance) of edge $e \in E$, where any node $v \in V$ is in T or is adjacent to a node in this tree. Link cost distances are related as usual [24] with the power required to establish the topology of the involved dominating tree connections. The topology cost of a tree T is the sum of the cost distances of its edges. The DT problem is known to be NP-Hard [24].

In some sensor networks, with each edge $\{u, v\} \in E$, of extremities $u, v \in V$, there is associated an uncertain power consumption d^{ξ}_{uv} necessary to maintain the data transmission between u and v operational, where ξ denotes a random variable of known probability distribution function. Since power consumption in each sensor may vary due to weather or radio interference, sometimes the available power (battery autonomy of sensors) to keep the network operational becomes insufficient. This may cause loss of communication among the sensors. To overcome this problem, in this paper we consider probabilistic constraints related to power consumption in the classic DT problem. The resulting probabilistically constrained dominating tree (PCDT) problem consists in finding a minimum cost distance dominating tree $T = (V(T), E(T))$ of G, with set of nodes $V(T)$ and set of edges $E(T)$ subject to a probabilistic constraint related to the risk that the total power consumption of each sensor $u \in V(T)$ exceeds a given limit d_m corresponding to its power autonomy. The power consumption of a sensor u is defined as the sum $\sum_{\{u,v\} \in E(T)} d^{\xi}_{uv}$ of all links in $E(T)$ connected to u in T.

Another classic optimization approach for planning sensor networks considers the minimum number of connected dominating sensors [11]. In this case, the problem is equivalent to the one in [4,24] when the distance between sensors equals 1. The novelty here is to provide new ideas for the development of communication protocols for sensor networks by considering both the impact of distances among sensors and the risk that power consumption of sensors exceeds a power threshold available to keep the network operational. In this sense, our work is a first attempt to answer this question by considering jointly or separately dependence among power consumption of the links of the network in a unified framework.

Both the stochastic mathematical optimization model and the theory to make it tractable by mixed integer linear programming are contributions to future algorithm developments for practical applications. In this sense, this step of our research focuses mainly on the theory rather than implementing and evaluating the over-

* Corresponding author.
E-mail addresses: pablo.adasme@usach.cl (P. Adasme), rca@lia.ufc.br (R. Andrade), abdel.lisser@lri.fr (A. Lisser).

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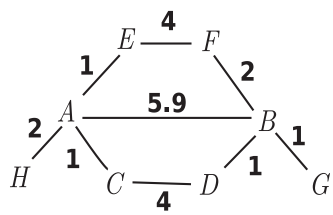

Fig. 1. Example of an undirected connected sensor network.

head of the proposed models in practice. Notice however, that any dominating tree topology proposed as solution by our model can be used in real implementations as well as in [4,11,24]. The reader is referred to [8] for further details on the involved problems in sensor networks.

1.1. An application of a dominating tree sensor network

In Fig. 1, we show an example of an undirected connected sensor network. The links of the graph in Fig. 1 represent the possible data exchange among the 8 sensors $\{A, B, \ldots, H\}$. Assume that two sensors connected by a link are at a unit distance of each other and that values near each communication link represent, deterministically the power consumption required to keep each link operational. The objective is to determine the minimum distance dominating tree topology for the communication among sensors respecting the power autonomy of each sensor. In fact, if the power autonomy of each sensor is unlimited, then the optimal dominating tree topology should be the unique link $T = (\{A, B\})$ of unit distance, since minimizing the number of connected dominating nodes solves this problem [11]. However, if with each sensor we associate a battery autonomy limit of 5 units to be used with all its communication links, then $T = (\{A, B\})$ is no more feasible because the power consumption required by this link is equal to 5.9 units and this value exceeds the available power for sensors A and B. By inspection, the optimal solution for this example is the dominating tree topology $T' = (\{A, C\}, \{C, D\}, \{B, D\})$, of distance equal to 3, with total link power consumption of sensors A, B, C, and D being equal to 1, 1, 5, and 5, respectively. Of course, we assume that the remaining nodes not in T' will communicate with their nearest neighbors in T' and that the required power to accomplish this task is negligible for the problem.

Thus, in order to deal with the probabilistic constraints when power consumption of the existing links vary according to, e.g., extreme and uncertain weather conditions, we propose a polynomial compact formulation for the DT problem which is based on dominating arborescence of a directed graph and give a proof of its correctness. The compact model allows us to obtain deterministic equivalent mixed integer linear programming formulations for the PCDT problem. Numerical results show that the new approach allows us to solve to optimality random and benchmark instances for the DT and PCDT problems with up to 120 nodes. To the best of our knowledge, this is the first work reporting proven optimal solutions for challenging benchmark instances of the classic DT problem [4,26]. Moreover, the probabilistic modeling approach allows us to handle joint and separate probabilistic constraints in a unified framework.

1.2. Probabilistic constraints

Stochastic programming is a powerful technique to deal with the uncertainty of some input parameters of a mathematical programming problem [23]. These parameters are assumed to behave as random variables which are distributed according to a given probability distribution function. In particular, the probabilistically constrained approach imposes a probability of occurrence for some (or all) of the constraints of a mathematical model. This means that some of the constraints will be satisfied, at least for a given percentage [1,5,10,15,21,22]. As far as we know, no attempt has been made to deal with probabilistic constraints in the problem of designing sensor networks with a dominating tree topology under uncertain power consumption. Probabilistic constraints can be considered either separately [1,3,5,10,15,16,19–22] or jointly [5–7,17].

In this paper, we consider both joint and separate probabilistic constraints for the PCDT problem. For this purpose, we assume that the row vectors of the matrix (d_{ij}^{ξ}) are joint dependent multivariate normally distributed vectors with known means and covariance matrices. The dependence of the random vectors is handled by means of copulas [6,13]. A copula is a distribution function β: $[0, 1]^{|V|} \rightarrow [0, 1]$ of some $|V|$-dimensional random vector whose marginals are uniformly distributed on [0, 1]. We apply useful properties of Gumbel–Hougaard copulas to describe the dependence between the rows of the matrix (d_{uv}^{ξ}). The latter allows us to handle joint and separate probabilistic constraints in a unified framework. The theory of copula distributions was developed in probability theory and mathematical statistics [13,18]. Copula distributions allow us to find deterministic convex reformulations for probabilistic constraints while simultaneously handling dependent random variables. In the stochastic programming field, this has not been well developed yet. In fact, this is the most important reason for using copula distributions so far. Several copula distributions have been proposed such as Clayton, Gumbel and Frank copulas, among others. In this paper, we mainly focus on the use of Gumbel–Hougaard copula as it is the one that leads to more tractable deterministic reformulations as shown in [6], and also because it allows a unified framework to handle joint and separate probabilistic constraints simultaneously. In Section 4, we give a brief formal description of this copula.

In [13], the authors use copulas to come up with convexity results for chance constrained problems with dependent random right hand side whereas in [6] the authors apply Gumbel–Hougaard copulas to a quadratic optimization problem subject to joint probabilistic constraints. In this paper, we transform the probabilistic constraints into equivalent deterministic second order conic constraints that we further linearize. Consequently, we obtain deterministic equivalent mixed integer linear programming (MILP) formulations for the PCDT problem. Additionally, we investigate the impact of using valid inequalities referred to as generalized sub-tour elimination constraints in all our proposed models [11].

The paper is organized as follows. In Section 2, we introduce both separate and joint probabilistic constraints for the problem. Subsequently, we introduce a new compact polynomial formulation for the DT problem and give a proof of its correctness. Then, in Sections 3 and 4 we present deterministic equivalent MILP formulations using separate and joint probabilistic constraints, respectively. Then, in Section 5 we conduct numerical experiments in order to compare all the proposed models for random and benchmark [26] instances for the DT and PCDT problems. Finally, in Section 6 we conclude the paper and provide some insights for future research.

2. Problem formulation

In this section, we present a new variant of the classic DT problem while considering both separate and joint probabilistic constraints that we incorporate in the proposed DT model leading to the PCDT problem formulation. Then, we introduce a compact polynomial formulation for the problem which is based on arborescence of directed graphs and give a proof of its correctness.