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Sparse channel estimation in OFDM systems using compressed sensing techniques in a Bayesian framework☆

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ABSTRACT

Compressed Sensing (CS)-based sparse channel estimation in a Bayesian framework for Orthogonal Frequency Division Multiplexing (OFDM)-based communication systems is presented in this paper. An OFDM signal model having interference free region of the received training sequence is developed. The significance of Bayesian approach in the formulation of an estimator is shown by Bayesian Bound Analysis. Based on the developed signal model, the interference-free region of the received OFDM signal is used for sparse channel estimation, utilizing CS reconstruction algorithms and prior statistical knowledge of channel. The proposed CS-based channel estimation method in the statistical framework results in a low complexity estimator, where the received samples used for estimation are less than that required for conventional techniques using Maximum Likelihood and Maximum *a posteriori* methods. The estimation methods are analyzed by numerical simulations and are found to have better performance when compared with previous algorithms.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has been adopted by numerous wireless communication systems due to its high spectral efficiency and robustness to multipath fading. The performance of OFDM-based systems is deteriorated if the channel is not estimated and compensated properly [1,2]. Several algorithms have been introduced for the compensation of the degrading effects of the channel. The importance of the estimation of the channel in OFDM-based communication systems is described in [3] with the formulation of different estimation algorithms. Also, joint estimation of channel and impairments in the OFDM-based systems are explored in [4]. A joint channel estimation algorithm for OFDM systems has been proposed in [5].

Many multipath channels are characterized as sparse fading channels where the channel coefficients are sparse in time domain [6–8]. Compressed Sensing (CS) is a novel signal processing technique where a sparse signal can be estimated with fewer samples than usually required [9–11]. Therefore, sparse channel estimation is possible at the higher accuracy with less number of samples using CS techniques [7,9,12]. Furthermore, low complexity algorithms for the estimation of synchronization impairments with the sparse channel are developed in [13].[2]

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[2] Notations: Upper case bold letters denote matrices and lower case bold letters denote column vectors. $\hat{\mathbf{A}}$ denotes the estimate of **A.** $\mathbf{I}_M$ denotes an $M \times M$ identity matrix. identity matrix. $\mathbf{A}^H$ and $\mathbf{A}\dagger$ denote conjugate transpose, and pseudo-inverse of **A**, respectively. $[\mathbf{A}]_{m,\,n}$ denotes the $(m,\,n)^{\text{th}}$ element of

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But the methods described in [3–13] have neglected the prior statistical information related to channel in the formulation of joint estimators and are not optimal in a statistical approach. The prior knowledge of the channel can be extracted using the methods described in [14], where estimation techniques of Statistical Properties of Fading Channels are discussed. In [14], algorithms for Maximum Likelihood (ML) estimates of the covariance parameters of fading channels together with the noise variance of Additive White Gaussian Noise (AWGN) channel are described. Therefore, the estimated AWGN noise variance and fading covariance matrix can be incorporated into channel estimation by utilizing the Bayesian estimators [15], which outperforms the classical channel estimators. Usually, the Mean Square Error (MSE) of the estimator is compared with Cramér-Rao Lower Bound (CRLB) [15], to assess the performance of estimation techniques for deterministic parameters. CRLB for joint estimation of the channel in OFDM-based communication system channel estimator is derived in [3,16]. The CRLB for estimation techniques including random parameters are derived using the Bayesian approach with prior statistical information and is called Bayesian CRLB (BCRLB) [17]. BCRLB for the estimation of phase offset in a single carrier communication system is derived, assuming a Wiener noise model in [18]. Joint estimation of random impairments, phase noise and channel, in OFDM system is explored in [19]. Theoretical analysis of OFDM channel estimation is given in further works [20,21] using current and previously received signal vectors.

The main contributions of this paper are listed below

1. CS-based channel estimation in OFDM systems using Bayesian framework is investigated.
2. An OFDM signal model having interference free region of the received training sequence is formulated.
3. The significance of Bayesian approach in the formulation of estimator is shown by comparing BCRLB and CRLB.
4. Based on the derived signal model, CS reconstruction algorithm is used to estimate the positions of non-zero coefficients of the sparse channel and using the estimated channel positions, a Bayesian algorithm for the estimation of channel coefficients is formulated.
5. The proposed algorithm is found to have lower computational complexity when compared with conventional techniques.

Organization of the paper

The remainder of the paper is organized as follows. The signal model formulation is described in Section 2. The BCRLB analysis of channel estimation in done in Section 3, followed by the formulation of CS-based estimation using Bayesian approach for OFDM in Section 4. Finally, the simulation results and its discussions are presented in Section 5.

2. Signal model

OFDM system using training sequence as guard interval is considered here. The input symbols undergoes Inverse Fast Fourier Transform (IFFT) operation and training sequence addition before being sent from the antenna at the transmitter. The transmitted signal is affected by fading channel, where the channel is modeled as exponentially decreasing independent Rayleigh multipath slow fading channel [22]. The Channel Impulse Response (CIR) is given by $\mathbf{h} = [h_0, h_1, \ldots, h_{P-1}]^T$, where h_k represents the k^{th} channel coefficient, and P denotes CIR length. The channel coefficients, h_k, are assumed to be distributed as $h_k \sim \mathcal{CN}(0, \sigma_k^2)$, $\Re\{h_k\} \sim \mathcal{N}(0, \sigma_k^2/2)$, and $\Im\{h_k\} \sim \mathcal{N}(0, \sigma_k^2/2)$. Therefore, the samples of the time-domain channel, $\mathbf{h}$, is circular Gaussian distributed as $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$, where $\mathbf{\Sigma}$ of $\mathbf{h}$, the diagonal covariance matrix, is assumed to be known [14]. Also, $\mathbf{h}$ is assumed to be a K-sparse signal [13], which implies that it contains K non-zero elements.

The guard interval, G, should at least be equal to $(P-1)$, where P is the CIR length, to completely mitigate Inter-Symbol Interference (ISI) [4]. The proposed algorithms in this paper are based on the fact that $G > (P-1)$. Therefore, the transmitted OFDM signal is represented as

$$[\mathbf{x}]_{(G+M)\times 1} = [d_0, d_1, \ldots, d_{G-1}, s_0, s_1, \ldots, s_{M-1}]^T \tag{1}$$

$$= [\mathbf{d} \;\; \mathbf{s}]^T, \tag{2}$$

where $[\mathbf{d}]_{1\times G}$ represents the training sequence and $[\mathbf{s}]_{1\times M}$ represents the transmitted data.

The received signal, after sampling and detecting the start of frame, is represented as

$$r(n) = \sum_{k=0}^{P-1} h_k x(n-k) + w(n), \tag{3}$$

A. $\otimes$ represents Kronecker product. *diag*[$\mathbf{x}$] represents a diagonal matrix having the elements of $\mathbf{x}$ as diagonal elements. Tr$(\mathbf{A})$ represents the trace of $\mathbf{A}$. Calligraphic letter $\mathcal{T}$ denotes set and $\mathcal{T}^c$ denotes set complement. $\mathbf{A}_{\mathcal{T}}(\mathbf{A}_{(\mathcal{T})})$ denotes the column (row) sub-matrix of $\mathbf{A}$ formed by the columns (rows) of $\mathbf{A}$ listed in the set $\mathcal{T}$. $\Im(\mathbf{A})$ and $\Re(\mathbf{A})$ denote the imaginary and real parts of $\mathbf{A}$, respectively. $\|\mathbf{y}\|_n$ denotes l_n-norm of $\mathbf{y}$. $\frac{\partial(\mathbf{B})}{\partial \eta}$ represents the partial derivative of $\mathbf{B}$. $p(.)$ denotes pdf and $\mathbb{E}_{ab}[.]$ denotes the expectation over a and b. Δ represent partial derivative operator, where $\Delta_{\boldsymbol{\gamma}} = [\frac{\partial}{\partial \gamma_1}, \frac{\partial}{\partial \gamma_2}, \ldots]^T$, and $\Delta_{\boldsymbol{\gamma}}^{\boldsymbol{\alpha}} = \Delta_{\boldsymbol{\gamma}} \Delta_{\boldsymbol{\alpha}}^T$.

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