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Finding poles and zeros in an analog circuit directly from its conductance matrix using eigenvalues☆

Reza Hashemian

Life Senior Member, IEEE, United States

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ABSTRACT

The article presents a state space approach to identify and extract poles and zeros in an RC circuit. The results obtained are shown to match with circuit simulation (WinSpice). A new procedure is also developed that allows construction of the state matrix *A* directly from the circuit inspections. This is proven to be a very efficient technique for identifying the state matrix *A* without going through a traditional nodal analysis. As shown, the matrix *A* is constructed by making specific measurements on the circuit itself, or by simulating solely the resistive circuit. Finally, a new method is presented, through which the zeros of a transfer function are first converted into poles and then the poles are extracted through the circuit eigenvalues. In addition, a new Modified Nodal Procedure (MNP) is presented in the Appendix that allows circuits with all types of independent and controlled sources. It shows how the presence of these sources are incorporated into the conductance matrix, providing a unified nodal (branch) solution to the circuit problem.

1. Introduction

ACCURATELY determining the response of an analog circuit to AC input signals with different frequencies has a long research history, in circuit theory. Conventional circuit simulators perform the task numerically by going into procedures that implement discrete (forward/backward) integration when dealing with storage elements such as capacitors, and inductors. In this technique the ordinary differential equations are first turned into algebraic equations that are representative of a resistive circuit in each step of the integration [1,2], and the results are then obtained by going through an ordinary nodal analysis in each step. This method of discrete integration is shown to be accurate enough for short computational steps [3,4], and it is relatively fast, depending on the integration steps. As stated in [3] the method works as follows: "the ac analysis computes a dc operating point and all of the necessary small signal parameters, then sweeps all ac sources through a set of frequencies, computing the ac small signal response at each of those frequencies."

There are also other methods for determining the frequency responses of analog circuits that take closed form solution paths, such as symbolic approaches. These methods try to extract poles and zeros of a circuit transfer function by solving its denominator and nominator polynomials [5–7]. An alternative and well developed approach, however, is through circuit state equations, or rather circuit eigenvalues. This is a popular method that solves for circuit roots (poles and zeros) directly using nodal representation of the circuit [8,9]. This study concentrates on this methodology. Here, the strength comes from the fact that each independent storage element holds its value (voltage for capacitors and currents for inductors) long enough to be labeled as a state.

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Fax. 815-753-1289
E-mail address: reza@niu.edu

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Computationally, the eigenvalue approach uses the nodal admittance matrix to construct a state matrix A. Then by solving the characteristic equation Eq. (1), the eigenvalues of the circuit, λ_i, for all i, are determined [10–15].

$$\det(A - \lambda I) = 0 \tag{1}$$

It is shown that QR and QZ algorithms offer more efficient techniques in computing the eigenvalues of a circuit or system [3,4].[1] In addition, corresponding to each eigenvalue λ_i, Eq. (2) has at least one nontrivial solution for V_i, which is an eigenvector of A.

$$(A - \lambda_i I)V_i = 0 \tag{2}$$

Haley [8], for instance, proceeds with computation of poles of an RC circuit by separating conductance and capacitance matrices, G and C, in the nodal admittance equations, as in Eq. (3). This separation of the storage elements from those non-storage elements helps to identify the circuit poles more easily, and makes it straightforward to follow a symbolic approach to identify the circuit poles. It is shown that the procedure to find the poles is computationally simplified when the matrix C becomes diagonal in Eq. (3).

$$(sC + G)V = I \tag{3}$$

Hennig [5], on the other hand, uses capacitors as circuit ports and computes the port resistances to find the time constants in RC circuits. From here the real axis poles can simply be estimated. Similar procedures are employed by Constantinescu, Nitescu, Iordache, and Dumitriu1 [10], and also by Riaza and Tischendorf [12], where they define tree branches for the circuit capacitors and links for the inductors. The procedures produce both C and L as diagonal matrices, although the analytical details and the execution steps are mostly left out. Guerra, Rodriguez-Garcia, and Fernandez [9] make a comprehensive study on poles and zeros extraction using feedback to exchange pole and zeros. This is done to identify zeros.

First, the traditional approach is reviewed in this presentation. This includes the identification and extraction of poles and zeros of a circuit transfer function using the eigenvalue technique. The procedure is independently developed here to a full extended, and the results are compared with those obtained through a circuit simulator (WinSpice) for different examples. Next, a new method, still based on the eigenvalue problem, is developed that allows the state matrix A to be constructed directly from the circuit inspections. This is proven to be a very efficient and fast technique for identifying the state matrix A without going into any nodal circuit analysis. As it turns out, implementing this method requires some specific measurements done on the circuit variables (voltages and currents), and these measurements are only done on the resistive circuit. Finally, a new method is presented, through which the zeros of the original circuit are first turned into poles in a newly formed circuit, and then those poles are subsequently identified through the eigenvalue procedure.

To start with, a brief review of the traditional eigenvalue approach is presented for extracting the natural frequencies of RC circuits. The procedure can of course be extended to cover any active RLC circuit. The difference is that, in the latter case the inductances (as well as the mutual inductances) must be added to the storage elements of the circuit and count for new circuit states. In this situation both the capacitors, as the branches of a specified tree, and the inductances, as the tree links, are entered into the circuit analysis forming the state matrix A. However, due to the space limitation this portion of the analysis is not covered here.

The material in this article is arranged as follows. A rather brief review of the eigenvalue technique is presented in Section 2. This review starts with first specifying a circuit tree that contains all circuit capacitors as its branches. Then based on this tree a branch admittance matrix is constructed, which is subsequently contracted to generate a Reduced Branch Admittance Matrix (RBAM) for the circuit. Next, the RBAM is used to form the state matrix A, the solution of which results in identifying the eigenvalues, hence, the poles of the circuit. In Section 3 a new methodology in introduced that finds the state matrix A purely by inspection. The method is fast and avoids many computational steps including matrix inversion. Section 4 is devoted to finding the circuit zeros through eigenvalue technique. The section explains how zeros can be turned into poles through the use of Fixator Norator Pairs (FNP) and then extracted using state space methodology. Finally the conclusion is given in Section 5.

2. A review of eigenvalue procedure

Consider a linear RC circuit N, and select a tree t_c in N such that it contains all the circuit capacitors as tree branches. In case of a capacitor loop or a loop of capacitors plus some independent voltage sources, the loop is broken by applying one of the following procedures: 1) Apply the Δ to Y conversion to the loops, or 2) add a small resistor r_c in series with one of the capacitors in the loop. In either case an extra node is added to N, which causes a capacitor link to become a capacitor branch of t_c. The second method is adopted in this presentation because of its simplicity and accuracy. The problem with the first method is that, it creates a capacitor node[2]; and a capacitor node (or cut set) creates a DC charge trap, or simply a pole at the origin, which is often destabilizing. In the second method, however, the location of the extra pole created can be arbitrary selected on the s-plane (typically on the real axis), typically quite far away from the rest of the circuit roots. In

[1] QR and QZ are eigenvalue algorithms that find a given state matrix A as a product of two matrices Q and R (or Z,), where Q is an orthogonal matrix and R is an upper triangular matrix.

[2] A node that is connected to capacitors, only.