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Linguistic majorities with difference in support

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ABSTRACT

In social choice voting, majorities based on difference of votes and their extension, majorities based on difference in support, implement the crisp preference values (votes) and the intensities of preference provided by voters when comparing pairs of alternatives, respectively. The aim of these rules is declaring which alternative is socially preferred and to that, they require the winner alternative to reach a certain positive difference in its social valuation with respect to the one reached by the loser alternative. This paper introduces a new aggregation rule that extends majorities based on difference of votes from the context of crisp preferences to the framework of linguistic preferences. Under linguistic majorities with difference in support, voters express their intensities of preference between pairs of alternatives using linguistic labels and an alternative defeats another one when a specific support, fixed before the election process, is reached. There exist two main representation methodologies of linguistic preferences: the cardinal one based on the use of fuzzy sets, and the ordinal one based on the use of the 2-tuples. Linguistic majorities with difference in support are formalised in both representation settings, and conditions are given to guarantee that fuzzy linguistic majorities and 2-tuple linguistic majorities are mathematically isomorphic. Finally, linguistic majorities based on difference in support are proved to verify relevant normative properties: anonymity, neutrality, monotonicity, weak Pareto and cancellativeness.

1. Introduction

Decision making problems deal with the social choice of the best alternative among all the possible alternatives taking into account the views and opinions, i.e. the preferences, of all the individuals of a particular social group [10,34,38]. Two approaches are possible to address these problems [24,26]: a direct approach that derives a social choice from the sole manipulation and processing of the information provided by all the individuals without the intermediate derivation of any kind of collective information using a fusion or aggregation operator, which is characteristic of the indirect approach. Obviously, the type of aggregation rule implemented in the second approach is crucial in designing the corresponding social choice rule, and ultimately in the final social solution to the decision making problem. This paper deals with this specific issue, and it is devoted to the introduction of a new aggregation rule for individual preferences.

A comparison study between different alternative preference elicitation methods is reported in [32], where it was concluded that pairwise comparison methods are more accurate than non-pairwise methods. The main advantage of pairwise comparison methods is that facilitates individuals expressing their preferences because they focus exclusively on two alternatives at a time. Given two alternatives, an individual either prefers one to the other or is indifferent between them, which can be represented using a preference relation whose elements represent the preference of one alternative over another one. There exist two main mathematical models to represent pairwise comparison of alternatives based on the concept of preference relation [10,35]: in the first one, a preference relation is defined for each one of the above three possible preference states, which is usually referred to as a preference structure on the set of alternatives; the second one integrates the three possible preference states into a single preference relation. This paper deals with the second type of relations, for which reciprocity of preferences is usually assumed in order to guarantee the following basic rationality properties in making paired comparisons [37]: indifference between any alternative and itself, and asymmetry of preferences, i.e. if an individual prefers alternative x to y, that individual does not simultaneously prefer y to x.

In classical voting systems the set of numerical values $\{1, 0.5, 0\}$, or its equivalent $\{1, 0, -1\}$ [10], is used to represent when the first alternative is preferred to the second alternative, when both alternatives are considered equally preferred (indifference), and when the second alternative is preferred to the first one,

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respectively. This classical preference modelling constitutes the simplest numeric discrimination model of preferences, and it proves insufficient in many decision making situations as the following example illustrates: Let x, y, z be three alternatives of which we know that one individual prefers x to y and y to z, and another individual prefers z to y and y to x; then using the above numerical values it may be difficult or impossible to decide which alternative is the best. As Fishburn points out in [10], if alternative y is closer to the best alternative than to the worst one for both individuals then it might seem appropriate to 'elect' it as the social choice, whilst if it is closer to the worst than to the best, then it might be excluded from the choice set. Thus, in many cases it might be necessary the implementation of some kind of 'intensity of preference' between alternatives.

The concept of fuzzy set, which extends the classical concept of set, when applied to a classical relation leads to the concept of a fuzzy relation, which in turn allows the implementation of intensity of preferences [42]. In [2], we can find for the first time the fuzzy interpretation of intensity of preferences via the concept of a reciprocal fuzzy preference relation, which was later reinterpreted by Nurmi in [33]. In this approach, the numeric scale to evaluate intensity of preferences is the whole unit interval [0, 1] instead of {1, 0.5, 0}, which it is argued though to assume unlimited computational abilities and resources from the individuals [5].

Subjectivity, imprecision and vagueness in the articulation of opinions pervade real world decision applications, and individuals usually find difficult to evaluate their preference using exact numbers. Individuals might feel more comfortable using words by means of linguistic labels or terms to articulate their preferences [44]. Furthermore, humans exhibit a remarkable capability to manipulate perceptions and other characteristics of physical and mental objects, without any exact numerical measurements and complex computations [4,12,25,29,45]. Therefore, in this paper, the individuals' preferences between pair of alternatives will be assumed to be given in the form of linguistic labels.

It was mentioned before that the type of aggregation rule implemented is crucial in designing the corresponding social choice rule. This paper focuses on the majority voting rules, which are very easy to understand by voters and therefore, when comparing two alternatives, they are seen as very attractive and appropriate to aggregate individual preferences into a collective one. *Simple majority rule* [31] stands out among the different majority rules. Under this rule, an alternative defeats another one when the number of votes cast for the first one exceeds the number of votes cast for the second one. In fact, the requirement to declare indifference between two alternatives is quite strong given that both alternatives have to receive exactly the same number of favourable votes. Furthermore, under the simple majority rule, the support required for an alternative to be the winner is minimum because it is only required to exceed the defeated alternative in just one vote. Being the most decisive aggregation rule turns out to become a drawback because the collective decision is very unstable, i.e. it could be reverted with the change of just one vote. In an attempt to overcome this shortcoming, tougher requirements for declaring an alternative as the winner have been defined and studied. Among these rules, it is worth mentioning the following: *unanimous majority*, *absolute majority* and *qualified majorities* [10,9,36].

Majorities based on difference of votes (M_k) [14,27,23] constitute another general approach to majority voting rules. These majorities allow to calibrate the amount of support required for the winner alternative by means of a difference of votes fixed before the election process. At the extreme cases, i.e. no difference and maximum difference of votes, majorities based on difference of votes become the simple majority and unanimous majority, respectively. Moreover, if indifference is ruled out from individual preferences, they are equivalent to qualified majorities. With these rules, indifference between two alternatives is possible to be declared for more cases than under the simple majority rule. In fact, the indifference state could be enlarged as much as desired. The application of the majorities based on difference of votes to the case of [0,1]-valued reciprocal fuzzy preference relations is known as the *majorities based on difference in support* ($\widetilde{M_k}$) [15].

The aim of this paper is to fill the gap between majorities based on difference of votes and majorities based on difference in support by providing new majority rules based on difference of support in the linguistic framework. Linguistic majorities with difference in support keep the essence of the former rules in the sense that for an alternative to be declared winner a specific support fixed before the election is to be achieved. The challenge here is to formally generalise the rules to the case of being the preferences linguistic rather than numeric in nature. An additional challenge here is to relate the linguistic majorities with difference in support that can be obtained when the main two approaches to model and represent linguistic information are applied. On the one hand, linguistic preferences can be modelled using a cardinal approach by means of fuzzy sets and their associated membership functions [42]. On the other hand, an ordinal approach can be used to model and manage linguistic preferences using the 2-tuple symbolic representation [21]. Therefore, two new and different linguistic majorities with difference in support will be introduced: the *linguistic fuzzy majorities* (LM_K) and the 2-*tuple linguistic majorities* ($2TM_k$). Fig. 1 illustrates the new linguistic majorities in relation with the corresponding ones developed for numerical preferences.

The remainder of the paper is structured as follows: The next section introduces concepts essential to the understanding of the rest of the paper. Following that, Section 3 introduces the concept of linguistic majorities with difference in support and their mathematical formulation for the main two approaches to model and represent linguistic information: fuzzy set representation (Section 3.1) and the 2-tuple symbolic representation (Section 3.2). Section 4 proves that both linguistic majorities are mathematically isomorphic when fuzzy sets are defuzzified into their centroid. In Section 5, linguistic majorities based on difference in support are proved to verify the following relevant normative properties: anonymity, neutrality, monotonicity, weak Pareto and cancellativeness. Finally, in Section 6 conclusions are drawn and suggestions made for further work.

2. Preliminaries

Consider m voters provide their preferences on pairs of alternatives of a set $X = \{x_1, \ldots, x_n\}$. The preferences of each voter can be represented using a matrix, $R^p = (r^p_{ij})$, where r^p_{ij} stands for the degree or intensity of preference of alternative x_i over x_j for voter p. The elements of R^p can be numerical values or linguistic labels. In the following we focus on the former ones, leaving for Section 2.3 the second ones.

2.1. Numeric preferences

There are two main types of numeric preference relations: crisp preference relations and [0,1]-valued preference relations; with the second one being an extension of the first one, i.e. [0,1]-valued preference relations have crisp relations as a particular case.

1. A crisp preference relation is characterised for having elements r^p_{ij} that belong to the discrete set of values {0, 0.5, 1}. In this context, when alternatives are pairwise compared, voters declare only their preference for one of the alternatives or their indifference between the two alternatives. Thus, if $r^p_{ij} = 1$ then voter p prefers alternative x_i to alternative x_j, while if $r^p_{ij} = 0.5$ the