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Examining the eigenvalues effect to the computational cost in mobile robot simultaneous localization and mapping

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ABSTRACT

One of the biggest factors that contributes to the computational cost of extended Kalman filter-based simultaneous localization and mapping is the computation of the covariance update. This results from the multiplications of the covariance matrix with other parameters along with the increment of its dimension, which is twice the number of landmarks. This study attempts to look for an optimal solution to decrease the computational complexity of the covariance matrix without compromising the accuracy of the state estimation through eigenvalue approach. This paper presents a study on the matrix-diagonalization technique, which is applied to the covariance matrix in extended Kalman filter-based simultaneous localization and mapping to simplify the multiplication process. The behavior of estimation and covariance were observed based on four case studies to analyze the performance of the proposed technique.

1. Introduction

Simultaneous localization and mapping (SLAM) is one of the navigation techniques that enables the robot to move autonomously and observes its surrounding in an unknown environment. SLAM does not require a priori map, but with the aid of proprioceptive and exteroceptive sensors on board, the mobile robot is able to incrementally build a feature map of the environment and use this map to localize its position. The position of mobile robot and landmarks are determined by means of estimation method such as the Kalman filter [1], the particle filter [2] or the H_∞ filter [3]. These estimators will provide an estimation based on the measurement data that are recursively recorded by the sensors. SLAM has been applied in a wide range of mobile robot applications such as underwater, mining, unmanned air vehicles, and in home appliances [4–7].

Extended Kalman filter (EKF) has been widely used to solve estimation problems in SLAM due to the simplicity of the algorithm, its robustness and ability to apply the algorithm online compared to other approaches such as particle filter. However, the whole covariance matrix in EKF-based SLAM needs to be updated every time a new landmark is detected. This process involves a lot of mathematical operation, thus will increase the computational cost. Moreover, the dimension of covariance matrix will increase to twice the number of landmark, as more landmarks are detected. The classical EKF-based SLAM algorithm is known to have a cost of $\mathcal{O}(m^2)$, in which m denotes the number of landmarks within the map. This limits the use of EKF in a large environment (only a few hundred landmarks). Besides, the full-covariance structure is also very sensitive to the linearization errors, which will accumulate through time and may cause the divergence to the filter [8].

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Therefore, researchers have been trying to find the solution to mitigate the shortcomings either by (i) dividing the map into sub-local, local, and global map, (ii) using non-full SLAM, or (iii) focusing on the simplification of the covariance structure. This paper focuses on the third approach; the simplification of the covariance structure.

Guivant and Nebot [9] introduced a decorrelation algorithm to simplify the covariance matrix. The algorithm will decorrelate a subset of the states that is weakly correlated and cancel the weak cross-correlation terms in the covariance matrix. A positive semi definite matrix is added to the covariance matrix to reduce the computational and storage costs in SLAM. However, this technique has some drawbacks that lead to filter instability [10]. For that reason, the cross-correlation of the structure needs to be preserved [11]. A study was conducted to improve the technique through diagonalization of only a part of the covariance [12]. This technique is known as covariance inflation method, in which a pseudo-noise covariance is added to the covariance matrix to maintain the suboptimality of the filter, given that SLAM is considered as a partially observable system [13,14]. Besides covariance inflation, Julier and Uhlmann introduced a covariance intersection method for SLAM [8], a fusion technique that combines two covariances when the correlations between them are unknown, and this method has been implemented not only in SLAM, but also in other applications [15]. In this technique, the updating process is carried out in two independent steps; updating the robot, then updating the landmark. However, a new parameter ω exists in the algorithm that needs to be chosen through an optimization process.

This study was conducted to find an alternative technique in diagonalizing the covariance matrix of EKF-based SLAM. As an initial approach, the matrix will be diagonalized by finding its eigenvalues and rebuilding a new diagonal-covariance from these values. The results of the effect on the estimation and covariance behavior were presented, which have been obtained through simulations.

The remainder of this paper is structured as follows. Section 2 contains a brief explanation on the EKF-based SLAM models, the structure of covariance matrix and the technique of matrix diagonalization. Section 3 explains the diagonalization process based on four case studies. The simulated results are presented and discussed in Section 4. Finally, the conclusion is drawn in Section 5.

2. Problem formulation

Simultaneous localization and mapping process in a mobile robot is represented through discrete time dynamical system equation using process and observation model. Extended Kalman filter is used to estimate the position of the mobile robot and landmarks based on probabilistic approach. In this section, the formulation of the SLAM process and estimation through extended Kalman filter are explained generally. Detailed explanation on EKF-based SLAM can be found from [1,16]. Moreover, the technique of matrix diagonalization that will be implemented on the covariance matrix is also discussed.

2.1. Model of simultaneous localization and mapping (SLAM)

The process model in mobile robot SLAM describes the kinematics and movements of a mobile robot. The mobile robot moves in an environment and measures its relative distance to existing landmarks using sensors. This process is performed in order to locate its position and simultaneously detect and verify the position of the landmarks. On the other hand, the measurement process is represented using the observation model.

The process model of the mobile robot localization and mapping at time $k+1$, described as a function of state vector X_k, control input u_k and process noise w_k evaluated at time k, is defined as

$$X_{k+1} = f(X_k, u_{k+1}, w_{k+1}, k) \tag{1}$$

The state vector of a 2D SLAM $X_k \in \mathbb{R}^{3+2m}$ is a joint state-vector of robot position X_r and position of landmark X_m, which has the following structure

$$X_k = [X_r \quad X_m]^T \tag{2}$$

where the position of the mobile robot $X_r = [\theta_k \quad x_k^r \quad y_k^r]^T$ is represented by the robot heading angle θ_k and the coordinates of the center of mobile robot with respect to the global coordinate frame (x_k^r, y_k^r). The state of the landmarks $X_m = [\ \ell_1 \quad \ell_2 \quad \cdots \quad \ell_m\ ]^T$ are modeled as a set of point landmarks and described by the Cartesian coordinate (x_i, y_i), $i = 1, 2, \ldots, m$, where m refers to the number of landmarks in the environment. Therefore, the full state of the SLAM can be described as follows

$$X_k = \begin{bmatrix} \theta_k & x_k^r & y_k^r & x_1 & y_1 & \cdots & x_i & y_i \end{bmatrix}^T \tag{3}$$

The control input of the robot movement is designated by $u_k = [\ \gamma_k \quad \omega_k\ ]^T$, where γ_k is a mobile robot turning rate and ω_k is its velocity with associated process noises, $\delta\gamma$ and $\delta\omega$. The process noise w_k is a zero-mean Gaussian noise of $\delta\gamma$ and $\delta\omega$ with covariance Q_k, i.e. $w_k \sim \mathcal{N}(0,\ Q_k)$. Therefore, the whole process model for the complete system of mobile robot SLAM may be written as

$$X_{k+1} = F_k X_k + u_{k+1} + w_{k+1} \tag{4}$$

The mobile robot considered in this thesis is a two-wheel (uni-cycle) differential drive with the center of mass located below the axle of the robot, and is equipped with the onboard range and bearing sensors. The mobile robot is assumed