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An analysis of decomposition approaches in multi-objectivization via segmentation

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ABSTRACT

Multi-objectivization via Segmentation (MOS) has been shown to give improved results over other previous multi-objectivization approaches. This paper explores the mechanisms that make different segmentations in MOS successful in the context of the Traveling Salesman Problem (TSP). A variety of new segmentation methods are analyzed and theories regarding their performance are presented. Spatial segmentation methods are compared with other adaptive and static decomposition methods. Insight into why previous adaptive methods performed well is provided. New decomposition methods are proposed and several of these methods are shown to attain better performance than previously known methods of decomposition. The convergence of various degrees of multi-objectivization is examined leading to a new, more general segmentation algorithm, Multi-Objectivization via Progressive Segmentation (MOPS). MOPS combines the single-objective genetic algorithm with multi-objectivization in a general form. In a given run MOPS can progress from a more traditional single objective method to a strong multi-objectivization method. MOPS attempts to improve the ratio of fitness improvements to fitness decrements, signal-to-noise ratio (SNR), over the course of an evolutionary optimization based on the principle that often fitness improvements are generally easier to find early in the run rather than late in the run. It is shown that MOPS provides robust performance across a variety of problem instances and different computational budgets.

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1. Introduction

Multi-objectivization is a technique for solving single objective optimization problems. The technique reformulates the single objective problem into a multiple objective problem and then solves the reformulated problem using an Evolutionary Multi-objective Optimization (EMO) method [1]. Multi-objectivization is a relatively new optimization technique due in part to two reasons. Firstly, EMO methods are relatively recent and prior to the introduction of EMO methods in the late 1980s few efficient techniques existed to simultaneously find many solutions on the Pareto frontier. Secondly, and perhaps just as importantly, the size of optimization problems studied has increased to a point where multi-objectivization methods can be competitive. Small problems are generally not complex enough to require multi-objectivization techniques. Some research to date has provided weak evidence that larger problems benefit from more aggressive forms of multi-objectivization [2–4]. Additional stronger evidence is required.

Multi-objectivization techniques fall into two major categories [5]. The addition of novel objectives is one major approach. Novel objectives approaches have shown improved results over single objective optimization with the addition of objectives such as solution age [6], frame bar width [7], and the first derivative of the objective function [8]. The second major category of multi-objectivization is Multi-objectivization Via Decomposition (MVD). MVD divides the objective function into component objectives and then uses those objectives in the optimization process. MVD has been most commonly used on fitness functions that have a sum-of-parts property but has also been theorized as being useful in sum-of-product fitness functions [2].

Previous works on MVD have used two major approaches. The first approach, helper-objectives, utilizes the main objective in conjunction with additional decomposed objectives [2–4]. The second approach, *pure decomposition*, does not use the original problem's main objective but instead only works on the decomposed objectives [1,5,9,10]. Lochtefeld and Ciarallo [4] outlined several principles governing multi-objectivization via helper-objectives.

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Several of these principles are general and likely apply to multi-objectivization via pure decomposition. This paper studies the general principles that govern helper-objectives using a pure-decomposition method in order to determine the applicability of the principles governing helper-objectives on pure-decomposition approaches. These principles are studied in the context of the Traveling Salesman Problem (TSP).

The remainder of this paper is structured as follows. The background section focuses on multi-objectivization research to date. General principles of multi-objectivization are summarized. Further the TSP is described and prior research studying the TSP using multi-objectivization techniques is examined. The experiment section contains three distinct experiments. The first two experiments focus on finding and improving possible decompositions for the TSP by analysis of existing decompositions, proposed new decompositions, and empirical study of the performance of promising decompositions. The third experiment introduces MOPS and evaluates its performance against methods with a static degree of decomposition such as MOS. Finally, concluding remarks are provided that both summarize the work and recommend avenues for further research.

2. Background

The TSP is a classic combinatorial optimization problem that "is probably the most studied of *NP*-hard problems" [11]. The goal of the optimization is to find good or optimal low-cost tours that traverse a set of cities. A tour is a route that starts and ends at the same city and travels through each city exactly once. Frequently solutions for the TSP are defined by a permutation string which determines the sequence in which cities are visited. Each city appears in the string exactly once. TSPs have a multitude of practical applications and have been used in the past to model and solve problems related to applications in data clustering, drilling circuit boards, genome sequencing, and delivery and pickup [11]. Methods to solve large TSPs include heuristics such as Genetic Algorithms [12], Simulated Annealing (SA) [13], and tabu search [14], and exact methods such as branch and bound [11] and dynamic programming.

The remainder of this background section is composed of two areas. The first section, *multi-objectivization studies on the TSP*, summarizes previous work accomplished on the TSP. The second section, *principles of multi-objectivization*, describes known principles that apply to multi-objectivization techniques. For a broader background on multi-objectivization the reader is referred to [3,4].

2.1. Multi-objectivization studies with the TSP

Multi-objectivization has been studied with the TSP in at least three independent efforts under the same thread of research. Knowles et al. [1] examined the TSP using a multiple objective hill climbing algorithm. Later, Jensen [2] applied the concepts of helper-objectives to the TSP. Finally, Jahne et al. [10] studied the TSP and proposed a new method called Multi-Objectivization via Segmentation (MOS). These three works are discussed next.

Knowles et al. studied the TSP with multi-objectivization by pure decomposition using the Pareto-Envelope based Selection Algorithm (PESA) [15], a multi-objective hill climber. To turn the main objective into multiple objectives, two random cities, cities A and B, were selected. The decomposed objectives were the travel cost of moving from city A to city B, and the travel cost of moving from city B to city A. Since the full tour consists of going from city A, through some cities and on to city B, and then through some other cities and back to city A, the decomposed objectives ensured all costs associated with a full tour were considered. The multi-objectivization method using PESA outperformed its single-objective counterpart on six different TSPs ranging from 20 to 100 cities in size.

The Knowles et al. approach suffered from three weaknesses. Firstly, decompositions could be degenerate if cities A and B were close to each other. Secondly, and exasperating the first weakness, only a single decomposition was used in a given run which made the run heavily based on a single problem division. Lastly, identical solutions could be incomparable in the Pareto sense if a tour were reversed in two or more different solutions [2]. Identical, incomparable solutions in the Pareto sense can result in inefficient tracking of solutions by an EMO algorithm.

Jensen [2] corrected these weaknesses by explicitly assigning each city to two or more decomposed objectives. Jensen used the main objective simultaneously in conjunction with the decomposed objectives via a concept called helper-objectives. Because helper-objectives use the main objective simultaneously with the decomposed objectives, the best solution found would survive throughout the optimization. Since the objective function is based on the cost of travel between cities, each helper-objective summed the cost of incoming and outgoing links for its associated cities. This type of decomposition sums the cost of each link twice since links are shared between two cities. If two cities were adjacent in the tour and assigned to the same helper-objective, the cost of the links between the cities is added twice when calculating the objective value of the helper-objective. Similarly if adjacent cities are in different helper-objectives, the cost of the link is added to the objective value of both helper-objectives. Cities were randomly assigned to the different helper-objectives. To combat the possibility of a single, poorly-chosen decomposition, multiple random decompositions were used. These decompositions were used sequentially based on a random ordering. After a certain number of generations, a new set of helper-objectives would be used by the optimization. A more detailed description of helper-objectives is provided in [3]. Jensen studied 40 TSPs ranging from 99 to 2103 cities.

Jensen theorized that adaptive strategies using the decomposed objectives could give additional improved results. An adaptive strategy makes decisions about how the algorithm works based on the evolution of the population. Jahne et al. used adaptive decompositions of the cities based upon different properties of the costs of current links represented in the population [10]. The proposed MOS method uses pure decomposition, partitioning the cities into two decomposed objectives based upon a single dividing point. This dividing point is determined by examining a sample of individuals in the population to determine the representative cost of links associated with cities. Three different dividing points were considered. For instance, one decomposition used Expected Value Of Distances (EVOD) for each city to divide cities into two segments based upon above average and below average EVODs. If the represented cost of the links into and out of a city in the sample was greater than the average EVOD for all cities in sample, the city was assigned to the first segment. Conversely, if the represented cost was lower than the average EVOD in the sample, the city was assigned to the other segment. Decompositions that used more than two divisions were not studied because empirical evidence gathered by Jensen [2] on the Job Shop Scheduling Problem (JSSP) indicated the smallest (most basic) decompositions were the most competitive.

EVOD uses the expected value of the distance scores represented in a sample. A given distance score for a city is defined by summing all of the represented distances into and out of that city for the individuals in the sample. Suppose we need to calculate the distance score for city α for a sample of the population defined by the set I that contains ρ individuals. Let the function $P(i, \alpha)$ return the city immediately preceding city α in solution i. Similarly, let the function $S(i, \alpha)$ return the city immediately following city α in solution