Applied Soft Computing 19 (2014) 80–85

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Modeling project time–cost trade-off in fuzzy random environment

Hua Ke [a], Junjie Ma [b,*]

[a] School of Economics and Management, Tongji University, Shanghai 200092, China
[b] School of Law, Tongji University, Shanghai 200092, China

ARTICLE INFO

Article history:
Received 26 July 2011
Received in revised form 28 June 2012
Accepted 26 January 2014
Available online 12 February 2014

Keywords:
Project scheduling
Time–cost trade-off
Fuzzy random theory
Fuzzy random simulation
Genetic algorithm

ABSTRACT

In real projects, the trade-off between the project cost and the project completion time, and the environmental uncertainty are aspects of considerable importance for managers. For complex environment with more than one type of uncertainty, this paper presents three types of time–cost trade-off models, in which the project environment is described via introducing the fuzzy random theory. The expected value and the chance measure of fuzzy random variable are introduced for modeling the problem under different decision-making criteria. After that, this paper is devoted to designing a searching method integrating the technique of fuzzy random simulations and genetic algorithm for searching the quasi-optimal schedules. Finally, some numerical examples are given to demonstrate the effectiveness of the designed method for solving the proposed models.

1. Introduction

The time–cost trade-off problem studies how to modify project activities so as to achieve the trade-off between the completion time and the project cost, which is a specific type of the project scheduling problem. Kelley [1] first studied the problem. In the following years, the research on the time–cost trade-off problem most focused on the deterministic cases [2–4]. For solving the deterministic time–cost trade-off problem, the common analytical methods are linear programming and dynamic programming [5,6]. Besides, some heuristic algorithms, such as genetic algorithm (GA) [7,8], are also introduced.

Though much research work on the time–cost trade-off problem assumes the problem in deterministic environment, the real world is full of nondeterministic factors. For instance, the project completion time may vary for many external factors, such as the change of weather, the increase of productivity level and the use of additional manpower. Hence, much recent work introduced uncertain factors. The readers may refer to papers [9–11] to see the progress in stochastic project scheduling problem. In recent years, stochastic time–cost trade-off problem has also attracted much interest. Wollmer [12] discussed a stochastic linear time–cost trade-off problem, in which some discrete random variables were introduced. Gutjahr et al. [13] designed a modified stochastic branch-and-bound approach and applied it into a specific stochastic discrete time–cost trade-off problem. Laslo [14] described a stochastic critical-path-method time–cost trade-off model, including four fundamental formulations of the model and several new ideas for formulating the relationships between time–cost trade-offs. Zheng and Ng [15] presented a new approach for time–cost optimization model by integrating fuzzy set theory and nonreplaceable front with genetic algorithms, where fuzzy set theory was introduced to model the managers' prediction on activity times and costs as well as the associated risk levels. Zahraie and Tavakolan [16] embedded two concepts of time–cost trade-off and resource leveling and allocation in a stochastic multiobjective optimization model, where fuzzy set theory was applied to represent different options for each activity. Ke et al. [17] presented three stochastic time–cost trade-off models to meet different practical optimization requirements.

Sometimes the uncertainty in projects cannot be described by randomness. In fact, the activities of some projects may have been processed many times before, and with historical data, the uncertainty of activity duration time can be described by probability distributions. While the activities of some other projects may be short of statistical data, the duration times can be better described by fuzzy variables. With the development of the research on fuzziness, the fuzzy set theory was also applied into the project scheduling problem, originally by Prade [18]. To the best of our knowledge of the authors, the first work on the fuzzy time–cost trade-off problem was done by Leu et al. [19]. In Ref. [19], the activity durations were characterized by fuzzy numbers due to environmental variation, and the fuzzy relationship between the activity time and the activity cost was demonstrated by

* Corresponding author. Tel.: +86 21 65980458.
E-mail address: majunjie211@163.com (J. Ma).

http://dx.doi.org/10.1016/j.asoc.2014.01.040
1568-4946/

membership function. Furthermore, the philosophy of chance-constrained programming was introduced as the decision-making criterion. Jin et al. [20] gave a GA-based fully fuzzy optimal time–cost trade-off model, in which all parameters and variables were characterized by fuzzy numbers and an example in ship building scheduling was demonstrated. Eshtehardian et al. [21] built a multi-objective fuzzy time–cost model, in which fuzzy logic theory was introduced to represent accepted risk levels. Ghazanfari et al. [22] and Ghazanfari et al. [23] applied possibilistic goal programming to the time–cost trade-off problem to determine optimal duration for each activity in the form of triangular fuzzy numbers. Ke et al. [24] introduced credibility theory into this optimization problem and built three types of fuzzy models. Chen and Tsai [25] constructed membership function of fuzzy minimum total crash cost based on Zadeh's extension principle and transformed the time–cost trade-off problem to a pair of parametric mathematical programs.

Sometimes randomness and fuzziness may exist in the same real project. For instance, Ke and Liu [26] introduced the philosophy of random fuzzy programming into the project scheduling problem. For the complicated environment with randomness and fuzziness, fuzzy random programming is also a very powerful tool for solving time–cost trade-off problem. Fuzzy random variable was first introduced by Kwakernaak [27,28]. In this paper, we introduce the concept of fuzzy random variable initialized by Liu and Liu [29] for modeling time–cost trade-off problem in complicated uncertain environment. Three time–cost trade-off models are built, including the (α, β)-cost minimization model, the expected cost minimization model and the chance maximization model. A hybrid intelligent algorithm integrating fuzzy random simulation and GA is designed for solving the models.

The remainder of the paper is organized as follows: In Section 2, the fuzzy random time–cost trade-off problem is described, and the formulae of the project completion time and the project cost are deduced. In Section 3, based on some introduced concepts, three fuzzy random time–cost trade-off models are built. The degeneration of the models is also discussed in Section 3. For solving the established models, three types of fuzzy random simulations are designed and integrated with GA for designing a hybrid intelligent algorithm in Section 4. The following section illustrates the effectiveness of the hybrid intelligent algorithm by some numerical experiments. Finally, Section 6 draws some conclusions.

2. Problem description

For environmental or decision-making changes, the activity durations might vary, and meanwhile the activity costs also change correspondingly. For instance, hiring more workers might accelerate the project scheduling process to consequently save the project completion time and simultaneously increase the total project cost. Actually, in most real projects, the managers always need to take into account the trade-off between the total project cost and the project completion time. It is naturally desirable for the managers to find the most effective way to complete a project in some predetermined completion time limit and meanwhile with the 'minimal' cost in some sense.

Generally with the activity-on-arrow(AOA) network structure, a project can be described by a directed acyclic graph as illustrated in Fig. 1. Let $G=(V, A)$ be a directed acyclic graph representing a project, where $V=\{1, 2, \ldots, n\}$ is the set of nodes, A is the set of arcs representing the activities of the project.

First we introduce the parameter ξ_{ij} as a fuzzy random variable representing the normal duration time of activity (i, j) in A, whose uncertainty attributes to the variation of the external environment, and c_{ij} as the normal cost per day of activity (i, j), which is

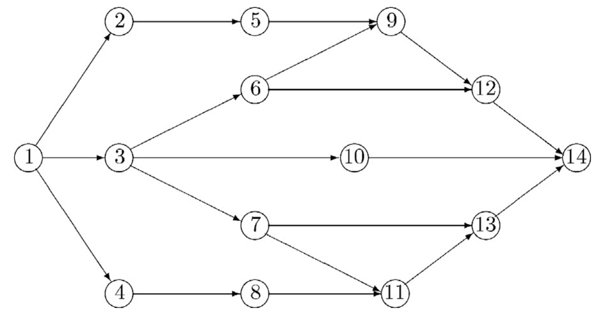

Fig. 1. A project.

a constant. That is, ξ_{ij} represents the duration time of activity (i, j) without the decision-making influence. The fuzzy random normal activity duration times are concisely written as $\boldsymbol{\xi}=\{\xi_{ij} : (i, j) \in A\}$. The decision variable x_{ij}, assumed to be an integer, represents the duration time change of activity (i, j) due to some manager decision-makings, such as hiring more or less workers, applying better or worse instruments, etc. To some extent, the concept is like the difference between normal duration time and crash duration time in project management. Owing to some practical limits, the variable x_{ij} is bounded by some interval $[l_{ij}, u_{ij}]$, where l_{ij} and u_{ij} are assumed to be integers. Accordingly for each activity (i, j), the extra cost d_{ij}, also assumed to be a constant, is defined as the cost change caused by the unit change of x_{ij}. With the project completion time requirement, it is necessary for the manager to find the optimal decision vector $\boldsymbol{x}=\{x_{ij} : (i, j) \in A\}$ to minimize the project cost.

We denote $T_{ij}(\boldsymbol{x}, \boldsymbol{\xi})$ as the starting time of activity (i, j) and the starting time of activity $(1, j) \in A$ is defined as $T_{1j}(\boldsymbol{x}, \boldsymbol{\xi})=0$. For simplicity, each activity is assumed to be processed only once all the foregoing activities are finished according to the precedence relationship type of finish-to-start linkage, and should be processed without interruption. With these assumptions, the starting time of activity (i, j), $i=2, 3, \ldots, n-1$, can be determined by

$$T_{ij}(\boldsymbol{x}, \boldsymbol{\xi}) = \max_{(k,i)\in A} \{T_{ki}(\boldsymbol{x}, \boldsymbol{\xi}) + \xi_{ki} + x_{ki}\}.$$

With the iterative consequence, the project completion time can be calculated by

$$T(\boldsymbol{x}, \boldsymbol{\xi}) = \max_{(k,n)\in A} \{T_{kn}(\boldsymbol{x}, \boldsymbol{\xi}) + \xi_{kn} + x_{kn}\}. \tag{1}$$

The project cost can be obtained by

$$C(\boldsymbol{x}, \boldsymbol{\xi}) = \sum_{(i,j)\in A} (c_{ij}\xi_{ij} - d_{ij}x_{ij}). \tag{2}$$

3. Fuzzy random time–cost trade-off models

3.1. Preliminaries

In this subsection, we will give some concepts of credibility theory and fuzzy random theory. The interested reader may refer to Liu [30] to see more of credibility theory and fuzzy random theory. For recalling the concept of fuzzy random variable, we first introduce the concept of fuzzy variable. Let Θ be a nonempty set, and $\mathcal{P}$ the power set of Θ.

Definition 1. (Liu and Liu [31]) The set function Cr is called a credibility measure if it satisfies:

(i) $\mathrm{Cr}\{\Theta\}=1$. *(ii)* $\mathrm{Cr}\{A\} \le \mathrm{Cr}\{B\}$ whenever $A \subset B$. *(iii)* $\mathrm{Cr}\{A\}+\mathrm{Cr}\{A^c\}=1$ for any $A \in \mathcal{P}$. *(iv)* $\mathrm{Cr}\{\cup_i A_i\} = \sup_i \mathrm{Cr}\{A_i\}$ for any $\{A_i\}$ with $\sup_i \mathrm{Cr}\{A_i\} < 0.5$.