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A simplified new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers

Ali Ebrahimnejad *

Department of Mathematics, Qaemshahr Branch, Islamic Azad University, Qaemshahr, Iran

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ABSTRACT

In a recent paper, Kaur and Kumar (2012) proposed a new method based on ranking function for solving fuzzy transportation problem (FTP) by assuming that the values of transportation costs are represented by generalized trapezoidal fuzzy numbers. Here it is shown that once the ranking function is chosen, the FTP is converted into crisp one, which is easily solved by the standard transportation algorithms. The main contribution here is the reduction of the computational complexity of the existing method. By solving two application examples, it is shown that it is possible to find a same optimal solution without solving any FTP. Since the proposed approach is based on classical approach it is very easy to understand and to apply on real life transportation problems for the decision makers.

1. Introduction

Transportation problem is an important network structured linear programming problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in the problem is to find the least total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling, and many others. In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way i.e., in crisp environment. However, in many cases the decision maker has no crisp information about the coefficients belonging to the transportation problem. In these cases, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and the fuzzy transportation problem (FTP) appears in a natural way.

Since the transportation problem is essentially a linear programming problem, one straightforward approach is to apply the existing fuzzy linear programming techniques [2,11,14,27,31,32,34,35] to the FTP. But, some of these techniques [2,32] only give crisp solutions which represent a compromise solution in terms of fuzzy data.

Several researchers have carried out investigations on FTP [1,4,5,12,13,15–26,28,29,33]. Zimmermann [35] developed Zimmermann's fuzzy linear programming into several fuzzy optimization methods for solving the transportation problems. Oheigeartaigh [25] proposed an algorithm for solving transportation problems where the supplies and demands are fuzzy sets with linear or triangular membership functions. Chanas et al. [4] investigated the transportation problem with fuzzy supplies and demands and solved them via the parametric programming technique. Their method provided solution which simultaneously satisfies the constraints and the goal to a maximal degree. In addition, Chanas and Kuchta [5] discussed the type of transportation problems with fuzzy cost coefficients and converted the problem into a bicriterial transportation problem with crisp objective function. Their method only gives crisp solutions based on efficient solutions of the converted problems. Jimenez and Verdegay [15,16] investigated the fuzzy solid transportation problem in which supplies, demands and conveyance capacities are represented by trapezoidal fuzzy numbers and applied a parametric approach for finding the fuzzy solution. Liu and Kao [24] developed a procedure, based on extension principle to derive the fuzzy objective value of FTP in that the cost coefficients and the supply and demand quantities are fuzzy numbers. Gani and Razak [12] presented a two stage cost minimizing FTP in which supplies and demands are as trapezoidal fuzzy numbers and used a parametric approach for finding a fuzzy solution with the aim of minimizing the sum of the transportation costs in the two stages. Yang and Liu [33] investigated the fixed charge solid transportation problem under fuzzy environment, in which the direct costs, the fixed charges, the supplies, the demands and the conveyance capacities are supposed to be fuzzy variables. Li

* Tel.: +98 09112129663.
E-mail addresses: aemarzoun@gmail.com, a.ebrahimnejad@srbiau.ac.ir

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et al. [22] proposed a new method based on goal programming for solving FTP with fuzzy costs. Lin [23] used genetic algorithm for solving transportation problems with fuzzy coefficients. Dinagar and Palanivel [10] investigated FTP, with the help of trapezoidal fuzzy numbers and applied fuzzy modified distribution method to obtain the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [26] introduced a new algorithm namely, fuzzy zero point method for finding fuzzy optimal solution for such FTP in which the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers. Kumar and Kaur [19] proposed a new method based on fuzzy linear programming problem for finding the optimal solution of FTP. Chakraborty and Chakraborty [3] proposed a method for the minimization of transportation cost as well as time of transportation when the demand, supply and transportation cost per unit of the quantities are fuzzy. Basirzadeh [1] proposed a systematic procedure for solving all types of FTP whether maximize or minimize objective function. Tao and Xu [30] established a rough multiple objective programming model for a solid transportation problem. Gupat and Kumar [13] proposed a new method to find solution of a linear multi-objective transportation problem by representing all parameters as interval-valued fuzzy numbers. Shanmugasundari and Ganesan [28] developed the fuzzy version of Vogel's and MODI methods for obtaining the fuzzy initial basic feasible solution and fuzzy optimal solution, respectively.

Chen [6] proposed the concept of generalized fuzzy numbers for situations that the membership function is not restricted to the normal form. Since then, a high number of researchers [7–9] have devoted their efforts to use generalized fuzzy numbers in real life problems, but there are few papers in which generalized fuzzy numbers are used to solve FTP. Kaur and Kumar [17] proposed a new method based on ranking function for solving FTP by assuming that the parameters of transportation problem are represented by generalized trapezoidal fuzzy numbers. In a recent paper in this journal, Kaur and Kumar [18] studied a special type of FTP by assuming that a decision maker is uncertain about the precise values of transportation cost only but there is no uncertainty about the supply and demand of the product. In that study, transportation costs were represented by generalized trapezoidal fuzzy numbers. They modified some existing methods for finding the initial basic feasible solution (IBFS) and fuzzy optimal solution using ranking function, in which transportation costs are represented as generalized trapezoidal fuzzy numbers. In this paper, we show that once the ranking function is chosen, the FTP is converted into crisp one, which is easily solved by the standard transportation algorithms. It is demonstrated that the method used in this study is simpler and computationally more efficient than the proposed method by Kaur and Kumar [18]. Since the proposed technique is based on classical approach it is very easy to understand and to apply on real life transportation problems for the decision makers.

This paper is organized as follows: In Section 2, formulation of special type of the FTP in terms of generalized trapezoidal fuzzy numbers and summary of the existing method are given. In Section 3, a simplified new method is proposed to find optimal solution of FTP. In Section 4, the application of the proposed method is shown using two application examples and the obtained results are discussed. Section 5 ends this paper with a brief conclusion and suggestion for future directions.

2. Fuzzy transportation problem

The FTP, in which a decision maker is uncertain about the precise values of transportation cost from the ith source to the jth destination, but sure about the supply and demand of the product, can be formulated as follows [18]:

$$\begin{aligned} &\min \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij} \\ &s.t. \sum_{j=1}^{n}x_{ij}=a_i, \quad i=1,2,\ldots,m, \\ &\sum_{i=1}^{m}x_{ij}=b_j, \quad j=1,2,\ldots,n, \\ &x_{ij}\geq 0, \quad i=1,2,\ldots,m, \quad j=1,2,\ldots,n. \end{aligned} \tag{1}$$

where a_i is the total availability of the product at ith source; b_j is the total demand of the product at jth destination; $\tilde{c}_{ij}$ is the approximate cost for transporting one unit quantity of the product from the ith source to the jth destination; x_{ij} is the number of units of the product that should be transported from the ith source to the jth destination or decision variables; $\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$ is total fuzzy transportation cost.

Let $\tilde{u}_i$ and $\tilde{v}_i$ be the fuzzy dual variables associated with ith row and jth column constraints, respectively, then the fuzzy dual of the FTP given in Eq. (1) will be as follows [18]:

$$\begin{aligned} &\max \sum_{i=1}^{m}a_i\tilde{u}_i \oplus \sum_{j=1}^{n}b_j\tilde{v}_j \\ &s.t. \tilde{u}_i \oplus \tilde{v}_j \precsim \tilde{c}_{ij} \quad i=1,2,\ldots,m, \quad j=1,2,\ldots,n. \end{aligned} \tag{2}$$

Kaur and Kumar [18] based on the ranking function given in Appendix A, introduced three methods namely generalized fuzzy northwest corner method (GFNCWM), generalized fuzzy least-cost method (GFLCM) and generalized fuzzy Vogel's approximation method (GFVAM) to find the initial basic feasible solution (IBFS) of the FTP given in Eq. (1). These methods are the direct extension of classical approaches. Then, they applied the generalized fuzzy modified distribution method (GFMDM) to find the fuzzy optimal solution of FTP given in Eq. (1) with the help of IBFS. The FGMDM is based on dual feasibility and complementary slackness theorem. In their proposed method all arithmetic operations are performed on the generalized trapezoidal fuzzy numbers, i.e., $\tilde{u}_i$, $\tilde{v}_j$ and $\tilde{c}_{ij}$. In the following section, we show that it is possible to find the same solution of the FTP given in Eq. (1) with the help of crisp transportation problem and so all arithmetic operations are done on real numbers instead of generalized trapezoidal fuzzy numbers.

3. Main results

According to ranking function given in Appendix A, it is possible to define a rank for each generalized trapezoidal fuzzy number. It follows that if $\tilde{A}_i=(a_i,b_i,c_i,d_i;w_i), (i=1,2,..,k)$ be k generalized trapezoidal fuzzy numbers, then $\Re(\tilde{A}_i)=w(a_i+b_i+c_i+d_i)/4$, where $w=\min\left\{w_i, i=1,2,\ldots,k\right\}$. This helps us to convert the FTP given in Eq. (1) into an equivalent crisp transportation problem. To do this, we substitute the rank of each generalized trapezoidal fuzzy number instead of the corresponding generalized trapezoidal fuzzy number in the FTP under consideration. This leads to an equivalent crisp transportation problem which can be solved by the standard transportation algorithms. Then, all arithmetic operations are done on the crisp numbers. As a result, the computational effort is decreased significantly in our proposed approach.

It is worth noting that the general steps of the Kaur and Kumar's method [18] to the FTP given in Eq. (1) are as follows:

Step 1. Find an IBFS of the FTP given in Eq. (1) using GFNCWM or GFLCM or GFVAM.