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Generalized fuzzy data envelopment analysis methods☆

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ABSTRACT

The conventional data envelopment analysis (DEA) measures the relative efficiencies of a set of decision making units (DMUs) with exact values of inputs and outputs. In real-world problems, however, inputs and outputs typically have some levels of fuzziness. To analyze a DMU with fuzzy input/output data, previous studies provided the fuzzy DEA (FDEA) model and proposed an associated evaluating approach. Nonetheless, numerous deficiencies must be improved, including the α-cut approaches, types of fuzzy numbers, and ranking techniques. Moreover, a fuzzy sample DMU (SDMU) still cannot be evaluated for the FDEA model. Therefore, the present paper proposes a generalized FDEA model which can evaluate SDMU and the traditional FDEA model. Five evaluation methods are provided and these methods not only improve the types of FDEA model, the types of fuzzy number, the α-cut approach but also firstly propose a new evaluation method based on vector. At last related algorithm and ranking methods are provided to test our new methods. A numerical experiment is used to demonstrate and compare the results with those obtained using alternative approaches.

1. Introduction

The data envelopment analysis (DEA) technique was introduced in the C^2R paper by Charnes et al. [1] (1978). Since its introduction, DEA has been developed by numerous studies. A large number of studies on DEA assume that both input and output data are crisp and have a specific case. In more general cases, the data for evaluation are often collected from investigations employing a polling approach, where in natural language, such as good, medium, and bad, are used to represent a type of general situation of the examined entities rather than a specific case. Thus, several studies proposed the fuzzy DEA (FDEA) model for input and output data [2]. However, there are still many places that need to be improved, such as the selected special point, the types of fuzzy number, the α-cut or α-level approach and the type of the FDEA model when evaluating FDEA model. Furthermore, the target decision making units (DMUs) of traditional DEA models are limited to internal DMUs that cannot evaluate a sample DMU (SDMU). To date, only a few studies have discussed the evaluation methods for an SDMU.

For the FDEA model, numerous evaluation approaches have been proposed. Meada et al. [3] formulated two DEA models. One model provides an upper limit (best case) efficiency, and the other model gives a lower limit (worst case) efficiency. Kao and Liu [4] also investigated fuzzy BCC models, and the idea behind their development is similar to that proposed by Meada et al., which falls into the parametric programming category. However, Kao and Liu called their approach the "α-cut" approach. In their approach the proposed membership function is LR fuzzy number. Guo and Tanaka [6] evaluated the fuzzy C^2R model using symmetric triangular membership function. In their method three different efficiencies are provided in different h-level and parameter t and they firstly proposed center efficiency value. However, some of the DMUs efficiency values are greater than 1 in Guo and Tanaka's approach. Saati et al. [5] studied the fuzzy C^2R model using asymmetrical triangular fuzzy numbers and they ranked the fuzzy DMUs according to the given α-cut. The basic idea was to transform the fuzzy C^2R model into a crisp linear programming problem using an alternative α-cut technique. Same as the Guo and Tanaka approach, some of the DMUs efficiency values in Saati et al's are greater than 1. Lertworasirikul et al. [7] analyzed the Fuzzy CCR model using trapezoidal fuzzy numbers, which introduced the idea of treating constraints as fuzzy events. This approach proposed fully efficient and α-possibilistic efficient definition according to that

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the efficiency value of the target DMU is equal or greater than 1 and they studied more cases (best–best, best–worst, worst–best, worst–worst) compared with the method proposed by Guo and Tanaka. At the same year Lertworasirikul et al. [8] studied fuzzy BCC model with normal or convex fuzzy numbers using credibility approach. Wang et al. [9] proposed a minimax regret-based approach (MRA), which is introduced to compare and rank the efficiency interval of DMUs. Punyangarm et al. [11] proposed fuzzy stochastic DEA model based on credibility approach and summarized previous approaches. León et al. [10] investigated the FDEA model using the L-R fuzzy number. They developed certain fuzzy versions of the classical DEA model (particularly, the BCC model) using several ranking methods based on a comparison of α-cut approach. Jahanshahloo et al. [12] also studied the FDEA model using a generalized L-R fuzzy number. They proposed a ranking method using l_1-norm. Wang et al. [13] studied fuzzy data envelopment analysis based upon fuzzy arithmetic and provide lower, middle and upper efficiency value of the target DMUs. Wen et al. [14] evaluated the FDEA model using a quasiconcave membership function. A fuzzy comparison of fuzzy variables was defined, and the C^2R model was extended into an FDEA model based on credibility measure. Hatami-Marbini et al. [15] proposed an ideal-seeking fuzzy data envelopment analysis framework. In this study, they presented a four-phase FDEA framework based on the theory of displaced ideal. The first step determined the best and worst relative efficiency of the target DMU, the second step determined the best and worst efficiency of DMU_j, the third step calculated the relative closeness of the DMUs and the last step ranked the DMUs according to their relative closeness. Furthermore, they studied more detailedly for the input and output data than other. Angiz et al. [16] proposed a four-stage approach based on DEA in the fuzzy environment to aggregate preference rankings. Angiz et al. [17] also introduced an alternative linear programming model that can include some uncertainty information from the intervals within the α-cut approach. Khoshfetrat and Daneshcar [18] studied fuzzy CCR model and proposed a new method for determining the lower bounds of fuzzy inputs and outputs. This improves the weak efficiency frontiers of the corresponding production possibility set. The author recently [19] proposed a fuzzy data envelopment analysis approach based on sample decision making units. In this paper the author firstly revealed the necessity of sample decision making unit model and improved the α-cut approach, the types of fuzzy number and the selected special point when evaluating FDEA model.

Besides the above study about the evaluating method of FDEA model, a large number of studies introduced the application of the FDEA model [20–27]. However, because of the complexity of the presented method it is difficult for most of practitioners to apply the method into the practical problems.

From the above analyses of the FDEA model, the following seven conclusions can be drawn:

- Most of the FDEA models that the researchers studied are special DEA models, such as CCR model BCC model.
- The membership functions of fuzzy numbers are assumed to be special membership functions, such as triangular, trapezoidal, L-R, and generalized L-R membership functions, and often, the same membership function for one FDEA model was used. In fact, there are many fuzzy numbers [28–33] besides the above membership function.
- The efficiency value of the fuzzy DMU was provided after replacing all the fuzzy DMUs with one of the DMUs of this fuzzy DMU. Most of the replaced DMUs are either the best or the worst DMUs of the fuzzy DMU.
- The procedures that solve the FDEA model used the same α-cut or parametric programming approach.
- The procedures that solved the FDEA used the ranking approach.
- The procedures that solved the FDEA used tolerance approach.
- The procedures that solved the FDEA used possibility approach.

The above-mentioned facts indicate that studies on FDEA models still focus on the special DEA model and fuzzy number and often apply only a single fuzzy number and the α-cut approach to one FDEA model. The ranking methods still have several limitations, and the selected DMUs, after applying α-cut, still remain as special DMUs. More important to the above conclusions is that these evaluation methods still cannot analyze a fuzzy sample DMU (SDMU). To address the above limitations of the FDEA model, the present paper proposes a generalized FDEA (GFDEA) model. The GFDEA model not only generalized the FDEA model but also contained the two key model-DEA models, which are the FDEA model with fuzzy sample target DMU and the FDEA model with sample target DMU. Five new evaluating methods are provided to both generalize the types of fuzzy numbers, the selected special point and the α-cut approach and to improve the traditional evaluating method of FDEA model. In these methods, $k-\lambda-$level $(0\le k,\lambda\le 1)$ efficiency methods not only contain most of the evaluating methods proposed by early study, but also firstly propose a new evaluation method based on vector. At last, related algorithm and ranking methods are proposed to illustrate our evaluating methods and demonstrate how our methods solve the practical problems.

The present paper is organized as follows: Section 2 introduces the FDEA models and their efficiencies. Section 3 discusses the proposed GFDEA model. Section 4 provides five evaluation methods for the GFDEA model. Section 5 presents the algorithm and ranking methods of GFDEA, and Section 6 gives numerical examples to demonstrate the concept. Section 7 discussed about the proposed method. Section 8 closes with the conclusion and future research directions.

2. Efficiency analysis of FDEA model

The most frequently used DEA model is the C^2R model, named after Charnes et al. [1]. The model assumes that there are n DMUs, each of which consumes the same type of inputs and produces the same type of outputs. Furthermore, the model supposes m to be the number of inputs and r as the number of outputs. All inputs and outputs are assumed to be nonnegative and at least one input and one output are positive. The following notations will be used throughout the current paper:

Notations

DMU_i ith DMU,
$FDMU_i$ ith fuzzy DMU,
DMU_0 target DMU,
$FDMU_0$ target fuzzy DMU_0,
SDMU target sample DMU,
FSDMU target fuzzy sample DMU,
$\boldsymbol{X}_i\in R^{m\times 1}$ column vector of inputs consumed by DMU_i,
$\boldsymbol{X}_0\in R^{m\times 1}$ column vector of inputs consumed by the target DMU,
$\boldsymbol{X}_{S0}\in R^{m\times 1}$ column vector of inputs consumed by the target SDMU,
$\boldsymbol{X}\in R^{m\times n}$ matrix of inputs of all DMUs,
$\boldsymbol{Y}_i\in R^{r\times 1}$ column vector of outputs consumed by DMU_i,
$\boldsymbol{Y}_0\in R^{r\times 1}$ column vector of outputs consumed by the target DMU,
$\boldsymbol{Y}_{S0}\in R^{r\times 1}$ column vector of outputs consumed by the target SDMU,
$\boldsymbol{Y}\in R^{r\times n}$ matrix of outputs of all DMUs,
$\boldsymbol{\lambda}=(\lambda_i)_{n\times 1}, \boldsymbol{\lambda}\in R^n$ column vector of a linear combination of n DMUs,
$\boldsymbol{\theta}$ objective value (efficiency) of the DEA model,
$\boldsymbol{\omega}\in R^{m\times 1}$ column vector of input weights,