Applied Soft Computing 13 (2013) 165–172

Contents lists available at SciVerse ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Soft computing based on new interval-valued fuzzy modified multi-criteria decision-making method

Behnam Vahdani*, R. Tavakkoli-Moghaddam, S. Meysam Mousavi, A. Ghodratnama

Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

ARTICLE INFO

Article history:
Received 30 July 2010
Received in revised form 26 May 2012
Accepted 4 August 2012
Available online 21 August 2012

Keywords:
Multi-criteria decision making (MCDM)
TOPSIS
Interval-valued fuzzy set (IVFS)
Robot selection

ABSTRACT

In this paper, a new interval-valued fuzzy modified TOPSIS (IVFM-TOPSIS) method is proposed that can reflect both subjective judgment and objective information in real life situations. This proposed method is based on concepts of the positive ideal and negative ideal solutions for solving multi-criteria decision-making (MCDM) problems in a fuzzy environment. The performance rating values and weights of criteria are linguistic variables expressed as triangular interval-valued fuzzy numbers. Furthermore, we appraise the performance of alternatives against both subjective and objective criteria with multi-judges for decision-making problems. Finally, for the purpose of proving the validity of the proposed method a numerical example is presented for a robot selection problem.

1. Introduction

The multi-criteria decision making (MCDM) provides an effective framework for comparison based on the evaluation of multiple conflicting criteria. The MCDM is one of the highest growing areas of operational research, as it is often realized that many concrete problems can be represented by several criteria. It was described as the most well-known branch of decision making [1]. The decision process of selecting a suitable alternative usually has to take many factors into considerations; for instance, organizational needs and goals, risks, benefits, limited resources, etc. Numerous qualitative and quantitative criteria may affect mutually when evaluate alternatives, which may make the selection process complex and challenging. In many cases, the decision maker (DM) has imprecise information about the alternatives with respect to an attribute. The classical MCDM methods cannot effectively handle problems with such imprecise information. These classical methods, both deterministic and random processes, are liable to be less effective in conveying the imprecision and fuzziness characteristics. This has led to the development of the fuzzy sets theory by Zadeh [2–5], who proposed that the key elements in human thinking are not numbers, but labels of fuzzy sets. The fuzzy set theory is a powerful tool to handle imprecise data and fuzzy expressions that are more natural for humans than rigid mathematical rules and equations [6–10]. It is obvious that much knowledge in real world situations is fuzzy rather than precise.

One of the well-known classical MCDM methods is the technique for order of preference by similarity to ideal solution (TOPSIS) developed by Hwang and Yoon [11]. The concept of this method is based on the chosen alternative that should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). In the classical MCDM methods (e.g., classical TOPSIS), the ratings and the weights of the criteria are known precisely. However, under many situations, crisp data are inadequate to model real-life situations since human judgments including preferences are often vague and cannot estimate his preference with an exact numerical value. A more practical approach may be to use linguistic assessments instead of numerical values. It is supposed that the ratings and weights of the criteria in the problem are assessed by means of linguistic variables. Hence, in this paper, a new interval-valued fuzzy set modified TOPSIS (IVFM-TOPSIS) method is proposed which could reflect both subjective judgment and objective information in real life situations. The proposed method is based on concepts of the positive ideal and negative ideal points for solving decision-making problems with multiple judges and multiple criteria in an interval-valued fuzzy environment. In this method, the performance rating values and the weights of criteria are linguistic variables expressed as interval-valued fuzzy numbers. Moreover, we appraise the performance of alternatives against subjective criteria via linguistic variables expressed as interval-valued fuzzy numbers. Finally, for the purpose of proving the validity of the proposed method, a numerical example is presented for a robot selection problem.

The remaining of this paper is organized as follows. In Section 2, we briefly introduce the original TOPSIS method. Section 3

---

* Corresponding author. Tel.: +98 21 82084183.
E-mail addresses: b.vahdani@ut.ac.ir, b.vahdani@gmail.com (B. Vahdani).

1568-4946/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.asoc.2012.08.020

illustrates interval-valued fuzzy sets. Section 4 describes the developed new interval-valued fuzzy modified TOPSIS (IVFM-TOPSIS) method to solve MCDM problems. Section 5 and Section 6 investigate a numerical example including an application to select a robot and discuss the proposed fuzzy modification decision method, respectively. Finally, the paper is concluded in Section 7.

2. TOPSIS method

The TOPSIS method is first proposed by Hwang and Yoon [11]. The PIS is a solution that minimizes the cost criteria and maximizes the benefit criteria whereas the NIS maximizes the cost criteria and minimizes the benefit criteria. The so-called benefit criteria are those for maximization, while the cost criteria are those for minimization. The best alternative is the first one, which is closest to the PIS and farthest from the NIS.

Suppose that a MCDM problem has m alternatives $(A_1, \ldots, A_m)$ and n decision criteria $(C_1, \ldots, C_n)$. Each alternative is evaluated with respect to n criteria. All the ratings assigned to the alternatives with respect to each criterion form a decision matrix denoted by $X = (x_{ij})_{m\times n}$. Let $W = (w_1, w_2, \ldots, w_n)$ be the relative weight vector about the criteria, satisfying $\sum_{j=1}^{n} w_j = 1$. Then, the TOPSIS method can be summarized as follows:

Step 1: Normalize the decision matrix $X = (x_{ij})_{m\times n}$ using the following equation:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n \tag{1}$$

where r_{ij} is the normalized criteria rating.

Step 2: Calculate the weighted normalized decision matrix $V = (v_{ij})_{m\times n}$:

$$v_{ij} = w_j r_{ij}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n$$

where w_j is the relative weight of the jth criterion or attribute, and $\sum_{j=1}^{n} w_j = 1$.

Step 3: Determine the positive ideal and negative ideal solutions by:

$$A^* = \left\{v_1^*, \ldots, v_n^*\right\} = \{(\max_i v_{ij} | j \in \Omega_b), (\min_i v_{ij} | j \in \Omega_c)\} \tag{2}$$

and

$$A^- = \{v_1^-, \ldots, v_n^-\} = \{(\min_i v_{ij} | j \in \Omega_b), (\max_i v_{ij} | j \in \Omega_c)\} \tag{3}$$

where Ω_b and Ω_c are the sets of benefit criteria and cost criteria, respectively.

Step 4: Calculate the Euclidean distances of each alternative from the PIS and the NIS, respectively:

$$D_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2}, \quad i = 1, 2, \ldots, m \tag{4}$$

and

$$D_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, \quad i = 1, 2, \ldots, m \tag{5}$$

Step 5: Calculate the relative closeness of each alternative to the ideal solution. The relative closeness of the alternative A_i with respect to A^* is defined by:

$$RC_i = \frac{D_i^-}{D_i^* + D_i^-}, \quad i = 1, 2, \ldots, m \tag{6}$$

Step 6: Rank the alternatives according to their relative closeness to the ideal solution. The bigger the RC_i, the better the alternative A_i is. The best alternative is the one with the greatest relative closeness to the ideal solution.

3. Interval-valued fuzzy sets

In the fuzzy sets theory, it is frequently difficult for an expert or DM to precisely quantify his or her opinion as a number in interval [0,1]. Therefore, it is more appropriate to represent this degree of certainty by an interval. Some researchers believed that the presentation of a linguistic expression in the form of fuzzy sets is not convincing [12]. Interval-valued fuzzy sets were recommended for the first time by Gorzalczany [13]. Also, Cornelis et al. [12] and Karnik and Mendel [14] noted that the main reason for proposing this new concept was the fact that in the linguistic modeling of a phenomenon, the presentation of the linguistic expression in the form of ordinary fuzzy sets was not clear enough. Wang and Li [15] defined interval-valued fuzzy numbers and gave their extended operations. Interval-valued fuzzy sets (IVFSs) have been widely used in real world applications; for instance, some of these applications are approximate reasoning [16,17], preference modeling [18], performance evaluation [19], mage filtering [20], uncertainty measure [21] and risk analysis [22].

Mustajoki et al. [23] utilized intervals in the SMART and SWING weighted methods. Halouani et al. [24] proposed two new multi-criteria 2-tuple group decision methods, called "Preference Ranking Organization Method for Enrichment Evaluation Multi-Decision maker 2-Tuple-I and II" (PROMETHEE-MD-2T-I and II). They integrated their procedure with both quantitative and qualitative information in an uncertain context. Opricovic and Tzeng [25] extended the VIKOR method for solving MCDM problems with conflicting and non-commensurable criteria, assuming that compromising was acceptable for conflict resolution, the decision maker wanted a solution that was the closest to the ideal, and the alternatives were evaluated according to all established criteria. Furthermore, this proposed method was compared with three multi-criteria decision-making methods, namely TOPSIS, PROMETHEE and ELECTRE. Zavadskas et al. [26] considered the application of grey relations methodology to defining the utility of alternatives, and offered a multi-criteria method of complex proportional assessment of alternatives with grey relations (COPRAS-G) for the analysis. In this model, the parameters of the alternatives were verified by the grey relational grade and expressed in terms of intervals.

Yao and Yu [27] utilized statistical data to derive level $(1 - \alpha, 1 - \beta)$ interval-valued fuzzy numbers to represent unknown alternative effectiveness scores. Then, by using the compositional rule of inference and signed distance to transform the fuzzy decision-making problem into crisp one, one can conveniently obtain the order of these different alternatives and subsequently obtain the best alternative. Dembczynski et al. [28] proposed dominance-based rough set approach (DRSA) to deal with multi-criteria classification (also called multi-criteria sorting, or ordinal classification with monotonicity constraints), where assignments of objects could be inconsistent with respect to dominance principle. The proposed methodology preserved well-known properties of rough approximations, such as rough inclusion, complementarity, identity of boundaries and precisiation. Furthermore, the