Applied Soft Computing 16 (2014) 20–33

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Differential evolution based 3-D guidance law for a realistic interceptor model

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ARTICLE INFO

Article history:
Received 20 December 2012
Received in revised form
12 September 2013
Accepted 29 November 2013
Available online 8 December 2013

Keywords:
Missile guidance
Optimal control
Optimal trajectory
Genetic algorithm
Online control
Real-time

ABSTRACT

This paper presents a novel, soft computing based solution to a complex optimal control or dynamic optimization problem that requires the solution to be available in real-time. The complexities in this problem of optimal guidance of interceptors launched with high initial heading errors include the more involved physics of a three dimensional missile–target engagement, and those posed by the assumption of a realistic dynamic model such as time-varying missile speed, thrust, drag and mass, besides gravity, and upper bound on the lateral acceleration. The classic, pure proportional navigation law is augmented with a polynomial function of the heading error, and the values of the coefficients of the polynomial are determined using differential evolution (DE). The performance of the proposed DE enhanced guidance law is compared against the existing conventional laws in the literature, on the criteria of time and energy optimality, peak lateral acceleration demanded, terminal speed and robustness to unanticipated target maneuvers, to illustrate the superiority of the proposed law.

1. Introduction

Interception of an aerial target by missiles or interceptors is an important practical problem. However, this is far from amenable to any easy solutions due to the inherent complexities of the problem. Realistic models of missile dynamics must include the effect of time-varying missile speed, thrust, drag and mass, besides gravity. The drag in turn is a function of the missile speed, atmospheric density, and lateral acceleration (latax). Another important practical constraint is the upper bound on the latax. Two simplifications resorted to in the literature are (i) consider only the kinematics, and (ii) separate the three dimensional (3-D) engagement into two planar (or two dimensional, or 2-D) kinematic models in the horizontal and vertical plane. Enabled by such gross simplifications, analytical or closed form guidance laws that can be easily implemented at every time instant, and with minimum computation, are formulated or derived, as in [56,38]. Given the much higher complexity of analysis and design in the 3-D space, guidance papers pertaining to 3-D engagements are far fewer in number, as compared to those for 2-D engagements, in spite of the fact that the first paper on 3-D guidance was published as early as in the 1950s itself [1].

Even for the simpler planar kinematic model, interception is not an easy task if the initial engagement geometry has high heading errors. Determination of the guidance law in a high heading error engagement, subject to the kinematic equation alone, has been the subject of many papers and techniques, and some of these techniques are based on methods of nonlinear control design. Some examples are: feedback linearization [2], relative heading error angle concept [22], state-dependent Riccati equation technique [9], extended proportional navigation [54], variable structure control [18,17], and, a combination of LOS rate and heading error [49]. Of these, only Cloutier and Stansbery [9] and Taur [49] deal with 3-D kinematic engagements; the rest consider only 2-D kinematic models.

The scenario becomes even more complicated if one imposes the additional requirement of optimality. Analytical optimal guidance laws derived from kinematic models require time-to-go parameter for their implementation, due to the free final-time nature of the problem. Consequently, accurate estimation of time-to-go itself has been another continuing topic of research, as in Ryoo et al. [35,36] and Tahk et al. [48].

The nonlinear optimal control problem applicable to high heading error initial geometries is a two point boundary value problem that is amenable to only numerical solutions and demand considerable computational effort, thereby not being implementable in

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http://dx.doi.org/10.1016/j.asoc.2013.11.017

real-time. Further, the performance of these numerical solutions deteriorates for even small deviations between the model assumed for design and the actual plant on which they are applied, due to the inherently open loop nature of these solutions. This makes the case for finding closed loop solutions or guidance laws, that are implementable in real-time, using a more realistic dynamic model, without resorting to the unrealistic simplifying assumptions for the sake of mathematical tractability.

The present paper formulates and studies the performance of an online-implementable optimal guidance law based on a realistic model of engagement that explicitly includes all the constraints mentioned in the beginning of Section 1, in 3-D physical space, for high initial heading errors. Except for treating the missile and target as point masses, no other approximations are made. The large number of inequality constraints in the formulation itself makes it very difficult to obtain an analytical or closed form solution, making numerical solution the only option. Any optimal control problem, linear or nonlinear, with any number of compatible equality and inequality constraints on state and controls can be solved by dynamic programming [28]. Another method of solution of the nonlinear optimal control problem is by converting it to a nonlinear programming (NLP) problem and solving the NLP problem, as in Cruz et al. [10]. A method that has been applied in optimal missile guidance, for a realistic model – though only 2-D – is Imado, Kuroda and Miwa [16]. However, as has been acknowledged sometimes by the authors themselves [16], all these are too computationally intensive for implementation in real-time.

In this paper, the optimal guidance problem is posed as a low dimensional polynomial optimization problem, and a numerical solution to this problem, computable in real-time, is obtained. The basic pure proportional navigation (PPN) law is augmented with a polynomial function of the heading error, and optimization is achieved by using DE to find the values of the coefficients of the polynomial. The proposed law is then compared with the all-aspect proportional navigation (AAPN) [39], and PPN laws on the following criteria: energy and time optimality, the peak latax demanded, terminal speed, and robustness to off-nominal conditions in the form of unanticipated target maneuvers. The AAPN law of Sim et al. [39] used the same structure as in the guidance law proposed in the present paper, but the solution proposed was applicable for the given initial conditions only, and needed to be recomputed for every different set of initial condition, as pointed out by the authors themselves. It also assumed the kinematic model of engagement. Moreover, the solution methodology was time consuming and hence unsuitable for online implementation. The law proposed in the present paper is applicable to any initial conditions, for a realistic dynamic model of engagement, and is also online implementable.

In the previous work of the authors [31], a DE based optimal guidance law was proposed for a two dimensional (2-D) kinematic model of missile–target engagement. It was a proof of concept that, for optimal missile guidance, evolutionary algorithms like the DE could serve as an alternative to the optimal control based methods, thereby circumventing the various difficulties posed by these methods. The work described in the present paper is much more elaborate, since it uses a 3-D realistic engagement model that includes missile dynamics and aerodynamic effects.

Compared to the previous work of the authors [31] which considered engagements with a simple, planar kinematic model, the present work assumes a 3-D realistic model that introduces the following complications: First, the state variables are subject to numerous interdependent equality and inequality constraints. Since these are indirect functions of the altitude, they depend on the vertical position of the missile in the inertial frame of reference. Secondly, the latax is applied by the missile in its own body frame of reference, and the LOS rates are measured and heading error angles are readily computed in the LOS frame of reference. This entails conversion from one frame of reference to another. Thirdly, unlike in the kinematic model in which the missile speed is constant, missile speed also enters the state as a variable, and is characterized by sharp discontinuities as the missile thrust switches from boost to sustain, and sustain to coast phases.

Fourthly, a very serious practical difficulty is that, with the dynamics also taken into account in the realistic model, the critical constraining factor is the thrust that is available only for a limited time period. The missile speed that is highly dependent on the thrust begins to fall off from the end of the boost phase, especially if the missile is still applying a large latax. Any practical interception is considered to be successful only if the missile speed is high enough just before interception, so that the missile is in a position to engage the target in the end game scenario. Hence, for a successful interception, the missile thrust should still be in the sustain phase, or at least not be too long into the coast phase.

The guidance law designed must take into account all these factors, and hence moving from a 2-D kinematic model to a 3-D realistic model is far from a mere extension of the guidance law designed for the 2-D kinematic model. The fifth and final complication that is of crucial importance is, the solution or the guidance law that involves the four complications described above, has to be available or computable in real time, to be of any practical relevance. The main contribution of this paper is in providing a real-time solution that is online implementable to this challenging problem, using differential evolution as the enabling method. Since the problem is a real world one, there is no generic, elegant solution; problem specific knowledge has to be incorporated, and real world complications have to be addressed, as described in the paper.

The paper is organized as follows: Section 2 presents the mathematical preliminaries, the 3-D PPN, and all-aspect proportional navigation (AAPN) laws, and the development of the 3-D counterpart of the 2-D differential evolution tuned all-aspect proportional navigation (DEPN-2D) law. Section 3 explains the DE algorithm used in the paper, Section 4 describes the design of the proposed 3-D guidance law using DE, Section 5 presents the simulation results and comparison of the proposed law with the other guidance laws, and is followed by concluding remarks in Section 6.

2. Vehicle models and guidance laws in 3-D engagement scenario

Using a Cartesian co-ordinate representation in an inertial frame of reference, the three dimensional (3-D) point-mass equations of motion for the missile and the target are modelled as follows:

Missile:

$$\begin{bmatrix} \dot{x_m} \\ \dot{y_m} \\ \dot{z_m} \\ \dot{\phi_m} \\ \dot{\gamma_m} \\ \dot{V_m} \end{bmatrix} = \begin{bmatrix} V_m \cos\gamma_m \cos\phi_m \\ V_m \cos\gamma_m \sin\phi_m \\ V_m \sin\gamma_m \\ a_{ym}/(V_m \cos\gamma_m) \\ (a_{pm} - g\cos\gamma_m)/V_m \\ (\Gamma - D)/m - g\sin\gamma_m \end{bmatrix} \tag{1}$$

$$a_{ym}^2 + a_{pm}^2 \le \overline{a}_m^2 \tag{2}$$