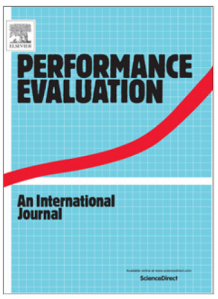

Analysis of a discrete-time single-server queue with an occasional extra server

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Analysis of a discrete-time single-server queue with an occasional extra server

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Abstract

We consider a discrete-time queueing system having two distinct servers: one server, the "regular" server, is permanently available, while the second server, referred to as the "extra" server, is only allocated to the system intermittently. Apart from their availability, the two servers are identical, in the sense that the customers have deterministic service times equal to 1 fixed-length time slot each, regardless of the server that processes them. In this paper, we assume that the extra server is available during random "up-periods", whereas it is unavailable during random "down-periods". Up-periods and down-periods occur alternately on the time axis. The up-periods have *geometrically distributed* lengths (expressed in time slots), whereas the distribution of the lengths of the down-periods is *general*, at least in the first instance. Customers enter the system according to a *general* independent arrival process, i.e., the numbers of arrivals during consecutive time slots are i.i.d. random variables with arbitrary distribution.

For this queueing model, we are able to derive closed-form expressions for the steady-state probability generating functions (pgfs) and the expected values of the numbers of customers in the system at various observation epochs, such as the start of an up-period, the start of a down-period and the beginning of an arbitrary time slot. At first sight, these formulas, however, appear to contain an infinite number of unknown constants. One major issue of the mathematical analysis turns out to be the determination of these constants. In the paper, we show that restricting the pgf of the down-periods to be a *rational* function of its argument, brings about the crucial simplification that the original *infinite* number of unknown constants appearing in the formulas can be expressed in terms of a *finite* number of independent unknowns. The latter can then be adequately determined based on the bounded nature of pgfs inside the complex unit disk, and an extensive use of properties of polynomials.

Various special cases, both from the perspective of the arrival distribution and the down-period distribution, are discussed. The results are also illustrated by means of relevant numerical examples.

Possible applications of this type of queueing model are numerous: the extra server could be the regular server of another similar queue, helping whenever an idle period occurs in its own queue; a geometric distribution for these idle times is then a very natural modelling assumption. A typical example would be the situation at the check-in counter at a gate in an airport: the regular server serves customers with a low-fare ticket, while the extra server gives priority to the business-class and first-class customers, but helps checking regular customers, whenever the priority line is empty.