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A queueing system for modeling cooperative wireless networks with coupled relay nodes and synchronized packet arrivals

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ABSTRACT

In this work we analyze a novel queueing system for modeling cooperative wireless networks. We consider a network of three saturated source users, say a central and two background users, two relay nodes and a common destination node. When the central user fails to transmit a packet directly to the destination, it forwards a copy of the blocked packet at both relay nodes in order to exploit the spatial diversity they provide. Moreover, each relay node receives also blocked packets from a dedicated background user. Relay nodes assist source users by retransmitting their blocked packets to the destination. Due to the complex interdependence among relays, and the wireless interference, the retransmission rate of a relay is affected by the state of the other. We consider a three-dimensional Markov process, investigate its stability, and study its steady-state performance using the theory of boundary value problems. Explicit expressions for the expected delay in the symmetrical model, and a generalization to $N > 2$ relay nodes are also given. Numerical examples are obtained and show insights into the system performance.

1. Introduction

Over the past few decades, wireless communications and networking have witnessed an unprecedented growth. The ever growing demands require high data rate and considerably large coverage areas. Relay-based cooperative wireless networks have been discussed as a possible solution for these future demands, since it is evidently proved that they allow a flexible exchange of data with great benefits on packet delay and energy consumption [1–5].

A cooperative wireless system operates as follows: There is a finite number of source users that transmit packets to a common destination node, and a finite number of relay nodes that assist source users by retransmitting their blocked packets; e.g., [2,4,6,7]. A cooperation strategy among sources and relays gives rise to space diversity protocols [5], under which, each user choose a number of "partners" (i.e., relays) that retransmit its blocked packets; e.g., [8–10]. We analyze a novel queueing system for modeling a cooperative network, by taking into account both the spatial diversity provided by the relays, and the effects on performance of the complex interdependence among relays due to the shared medium, as well as of interference [11–15].

In particular, when the central source user fails to transmit a packet to the destination node, it forwards its blocked packet at both relays (i.e., both relays overhear the transmission due to the wireless multicast advantage of the medium; two "partners"). Note that this feature gives rise to the so-called "fork-join" queues; [16,17]. On the contrary, a background source user cooperates only with a single relay node, and forwards its blocked packet only at that relay node. Note that due to the current trend towards dense wireless networks and the spatial reuse of resources (which in turn increase the impact

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of interference), it is essential to take into account the interdependency among transmissions in the network planning. Thus, the service rate of each relay node depends on the set of active relay nodes of the network (i.e., coupled processors). Such a characteristic is also common in cognitive radio (CR) users [4,18], and allows a node to exploit the silent (i.e., idle) periods of the other node, achieving the full utilization of the shared channel. For other applications of coupled processors in any system of servers in which the sharing of a common resource induces a correlation between their performance, see e.g., [11–15,19–21].

Our system is modeled as a three-dimensional Markov process, and we show that its steady-state performance is studied with the aid of the theory of boundary value problems [22]. The study of queueing systems using this theory was initiated in [23,24], and a concrete methodological approach was presented in [25]. For important generalizations see e.g., [8,26–32].

Our contribution. Besides its applicability, our work is also theoretically oriented. We provide for the first time, an exact analysis of a continuous time multiple access system that unifies the generalized two-demand model (i.e., fork-join queue), and the coupled processor model. It is evidently proved that each one of these systems is notoriously hard to study analytically; see [16,17,23,31–33]. Our model can be seen as a Markov modulated random walk in the quarter plane (M-RWQP), and its analysis leads to a matrix-form functional equation. However, due to its special structure, we can solve this matrix-form functional equation, and reduce it into a scalar functional equation corresponding to one state of the (modulated) chain. This scalar functional equation is treated using the theory of boundary value problems. Ergodicity conditions are also investigated. For the symmetrical system we obtain explicit expressions for the expected delay in each relay node, without solving a boundary value problem. We also investigate the model with $N > 2$ relay nodes (i.e. a central, and N background sources). Based on [34,35], we provide necessary and sufficient ergodicity conditions for the model with $N = 3$ relay nodes. Sufficient conditions for the model with $N > 3$ relay nodes are also given.

The paper is organized as follows. In Section 2 we present the model in detail, and form the fundamental functional equation. In Section 3 we proceed with a preparatory analysis, by investigating the kernel equation and the ergodicity conditions. Section 4 is devoted to the formulation and solution of two boundary value problems, which provide the generating function of the joint queue length distribution of the relays and destination node. Performance metrics are obtained in Section 5. In Section 6, we obtain explicit expressions for the expected delay at each relay node for the symmetrical system. A generalization to the model with $N > 2$ relay nodes is also given in Section 7. In Section 8 we obtain extensive numerical results that show insights into the system performance.

2. Model description and functional equation

We consider a network with three saturated source users, say S_i, $i = 0, 1, 2$, two relay nodes R_1, R_2 and a common destination node D. User S_i generates packets towards node D according to a Poisson process with rate λ_i, $i = 0, 1, 2$. Node D can handle at most one packet, that forwards outside the network. The service time of a packet at node D is exponentially distributed with rate μ.

Cooperation policy. Assume that relays have infinite capacity buffers and do not generate packets of their own, but instead, they only forward the packets they received from the source users. More precisely, if a direct transmission of a user's S_0 packet to the node D fails (i.e., the node D is busy), both relays store it in their queue, and try to forward it later (i.e., S_0 transmits within the overlapping area created by the intersecting covering regions of both relays). Thus, both relays overhear the transmission, and store a copy of the blocked packet in their buffers, i.e., the user S_0 exploits the spatial diversity provided by the relays; see [5,6]. Moreover, if a direct transmission of a user's S_i, $i = 1, 2$, packet to the node D fails, R_i stores it in its queue and try to forward it later (i.e. S_i cooperates only with R_i, $i = 1, 2$).

Retransmission policy. We consider coupled relay nodes, and thus, when both relays are non-empty, R_i retransmits a blocked packet to the node D after an exponentially distributed time with rate μ_i, $i = 1, 2$. If R_1 (respectively R_2) empties, then R_2 (respectively R_1) changes its retransmission rate from μ_2 (respectively μ_1) to μ_2^* (respectively μ_1^*); see Fig. 1. Note that a relay node is aware of the state of its neighbor, and accordingly, it changes its transmission parameters to allow more concurrent wireless communications.

Let $Q_k(t)$ be the number of stored packets in the queue of R_k, $k = 1, 2$, and $C(t)$ be the state of the node D respectively at time t. Clearly, $X(t) = \{(Q_1(t), Q_2(t), C(t)); t \geq 0\}$ is a continuous time Markov chain (CTMC) with state space $E = \{0, 1, \ldots\} \times \{0, 1, \ldots\} \times \{0, 1\}$. Define the stationary probabilities

$$\begin{aligned} p_{i,j}(n) &= \lim_{t\to\infty} P(Q_1(t) = i, Q_2(t) = j, C(t) = n) \\ &= P(Q_1 = i, Q_2 = j, C = n),\ i, j = 0, 1, 2, \ldots,\ n = 0, 1. \end{aligned}$$

Then, for $\lambda = \lambda_0 + \lambda_1 + \lambda_2$,

$$\begin{aligned} p_{i,j}(0)\left[\lambda + \sum_{k=1}^{2}\mu_k 1_{\{i,j>0\}} + \mu_1^* 1_{\{i>0,j=0\}} + \mu_2^* 1_{\{i=0,j>0\}}\right] &= \mu p_{i,j}(1), \\ p_{i,j}(1)[\lambda + \mu] = \lambda p_{i,j}(0) + \lambda_0 p_{i-1,j-1}(1)1_{\{i,j>0\}} + \lambda_1 p_{i-1,j}(1)1_{\{i>0\}} & \\ + \lambda_2 p_{i,j-1}(1)1_{\{j>0\}} + \mu_1 p_{i+1,j}(0)1_{\{j>0\}} + \mu_2 p_{i,j+1}(0)1_{\{i>0\}} & \\ + \mu_1^* p_{i+1,0}(0) + \mu_2^* p_{0,j+1}(0), & \end{aligned} \tag{1}$$