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Estimation of channel and Carrier Frequency Offset in OFDM systems using joint statistical framework

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ABSTRACT

Joint estimation of channel and Carrier Frequency Offset (CFO) in Orthogonal Frequency Division Multiplexing (OFDM) systems, using a Statistical framework, is shown in this paper. Hybrid Cramér–Rao Lower Bounds (HCRLBs) for the estimation of CFO together with the channel are obtained. The significance of prior information in the formulation of a joint estimator is shown by comparing HCRLB with the corresponding standard CRLB. We propose a Joint Maximum *a posteriori* (JMAP) algorithm for the estimation of channel and CFO in OFDM, utilizing the prior statistical knowledge of channel. To reduce the complexity of JMAP estimator, a Modified JMAP (MJMAP) algorithm, which has no grid searches, is also proposed. The estimation methods are analyzed by numerical simulations and resultant conclusions validate the better performance of the proposed algorithms when compared with previous algorithms.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has been adopted by numerous wireless communication systems due to its high spectral efficiency and robustness to multipath fading. OFDM based systems are susceptible to performance degradation due to impairments like Carrier Frequency Offset (CFO). CFO occurs due to the frequency differences between RF oscillators used in the OFDM transmitter and receiver, and channel induced Doppler shifts [1,2]. The CFO and channel deteriorate the performance of OFDM systems, if not estimated and compensated properly [3,4].

Several algorithms have been introduced for the compensation of the degrading effects of channel and CFO. Accurate estimates of symbol timing and carrier frequency offset using one training sequence is obtained in [5]. A Maximum Likelihood (ML) joint estimator for CFO together with Sampling Frequency Offset (SFO) for an OFDM system, assuming perfect channel knowledge, is described in [6]. The joint estimation of CFO and channel for OFDM systems in the presence of timing ambiguity is addressed in [7]. Also, joint estimation of CFO, SFO, channel, and timing error in an OFDM-based system is explored in [8]. Timing and frequency synchronization algorithms using nonlinear least squares estimation method are proposed in [9]. The authors of [10] presented a comparative study of different algorithms for frequency offset estimation in OFDM which are based on periodic training sequences. Furthermore, low complexity algorithms for the estimation of synchronization impairments with the sparse channel are developed in [11]. A joint ML time-frequency synchronization and channel estimation algorithm for MIMO-OFDM systems has been proposed in [12]. RLS-based joint estimation and tracking of the channel response, SFO, and CFO for OFDM is described in [13]. Further, joint pilot-aided estimation of the residual CFO and SFO in OFDM system is proposed in [14], with frequency estimates derived in closed-form. In [15], a pilot-aided joint Channel Impulse Response (CIR), CFO, and SFO estimation scheme, assuming a perfect start of frame detection and neglecting channel statistics, has been proposed for OFDM based systems.

But the methods described in [5–15] have neglected the prior statistical information related to channel in the formulation of joint estimators and are not optimal in a statistical approach. The prior knowledge of the channel can be extracted using the methods described in [16], where estimation techniques of Statistical Properties of Fading Channels are discussed. In [16], algorithms for ML estimates of the covariance parameters of fading channels together with the noise variance of Additive White Gaussian Noise (AWGN) channel are described. Therefore, the estimated AWGN noise variance and fading covariance matrix can be incorporated into channel estimation by utilizing the Bayesian estimators [17], which outperforms the classical channel estimators. Joint estimation of the random impairments in OFDM-based systems is investigated in [18], together with the derivation of bounds. Estimators

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based on ML and Minimum Mean Square Error (MMSE) criteria for the channel estimation are proposed in [19], without considering the effect of other impairments. Also, a Compressed Sensing based sparse channel estimation for OFDM-based communication systems is presented in [20]and [21].

Usually, the MSE of the estimator is compared with Cramér–Rao Lower Bound (CRLB) [17], to assess the performance of estimation techniques for deterministic parameters. CRLB for joint estimation of the channel in OFDM based communication system channel estimator is derived in [22]. The CRLB for estimation techniques including random parameters are derived using the Bayesian approach with prior statistical information and is called Bayesian CRLB (BCRLB) [23]. For estimations involving both deterministic and random parameters, the most suitable CRLBs are found to be Hybrid CRLBs (HCRLBs) [23].[2]

In this paper, we present the signal model formulation in Section 2 followed by the HCRLB analysis of estimation of channel together with CFO in OFDM in Section 3. The formulation of different MAP algorithms for the estimation of channel and CFO is given in Section 4. A MAP estimator for the channel in the absence of CFO is developed followed by a joint estimator for channel and CFO. A less complex joint estimator, which has no grid searches, is also proposed. Finally, simulation results are presented in Section 5.

2. Signal model

OFDM system using any constant modulus modulation technique is considered. Let T be the transmitter signaling period and M be the number of OFDM subcarriers with the subcarrier spacing given by $1/(MT)$. The input symbols undergo Inverse Fast Fourier Transform (IFFT) operation and cyclic prefix (CP) addition before being sent from the antenna at the transmitter. The transmitted signal is affected by fading channel, where the channel is modeled as exponentially decreasing independent Rayleigh multipath slow fading channel [24]. Frequency differences between carrier frequency oscillators used in the OFDM transmitter and receiver, and channel induced Doppler shifts cause a net CFO of Δf_c in the received signal, where f_c is the operating radio carrier frequency. The normalized CFO is defined as $\epsilon = \Delta f_c MT$, where M is the number of subcarriers and T is the sampling time.

The transmitted signal, $x(t)$, and received signal, $r(t)$, are given by

$$x(t) = \sqrt{2}\Re[x_b(t)\exp(j2\pi f_c t)], \tag{1}$$

and

$$r(t) = \sqrt{2}\Re[r_b(t)\exp(j2\pi f_c t)] \tag{2}$$

where $x_b(t)$ and $r_b(t)$ represent complex baseband equivalent representation of $x(t)$ and $r(t)$, respectively, and f_c is the carrier frequency. Assuming the wireless channel under consideration to be linear and time-invariant, the received signal, $r(t)$, can be expressed as

$$r(t) = \exp(j2\pi\Delta f_c t)\sum_{i=0}^{\infty} a_i x(t-\tau_i) + w(t). \tag{3}$$

where $x(t)$ is the transmitted signal, $w(t)$ is the AWGN process at the receive antenna, and a_i and τ_i represent overall attenuation and propagation delay, respectively, from the transmitter to receiver at the ith path. Thus, from (1)–(3)

$$\begin{aligned} r(t) &= \sqrt{2}\Re\left[r_b(t)\exp(j2\pi f_c t)\right] \\ &= \exp(j2\pi\Delta f_c t)\sqrt{2}\Re \\ &\quad\times\left[\left\{\sum_{i=1}^{\infty} a_i x_b(t-\tau_i)\exp(-j2\pi f_c\tau_i)\right\}\exp(j2\pi f_c t)\right]. \end{aligned}$$

Comparing with (2)

$$r_b(t) = \exp(j2\pi\Delta f_c t)\sum_{i=1}^{\infty} a_i^b x_b(t-\tau_i), \tag{4}$$

where $a_i^b = a_i\exp(-j2\pi f_c\tau_i)$. The baseband equivalent of input signal is represented as

$$x_b(t) = \sum_{l=0}^{\infty} x(l)sinc(t/T-l), \tag{5}$$

where $sinc(t/T) = \frac{\sin(\pi t)}{\pi t}$, $x(l)$ is the baseband discrete time transmitted signal, and T is the sampling time. Substituting (5) in (4), we get

$$r_b(t) = \exp(j2\pi\Delta f_c t)\sum_{i=1}^{\infty} a_i^b\sum_{l=0}^{\infty} x(l)sinc(t/T-\tau_i/T-l),$$

The sampled output at multiples of T are given by, $r(n) = r_b(nT)$, as

$$r(n) = \exp(j2\pi\Delta f_c nT)\sum_{l=0}^{\infty} x(n-l)h_l, \tag{6}$$

where h_l denotes the lth tap of Channel Impulse Response (CIR) given by

$$h_l = \sum_{i=1}^{\infty} a_i^b sinc[l-\tau_i/T].$$

The lth channel tap is interpreted as the $(lT)th$ sample of the baseband channel response $h_b(\tau)$ convolved with $sinc(\tau/T)$. Also, h_l is modeled as circular symmetric Gaussian random variable based on the assumption that there are a large number of statistically independent reflected and scattered paths with random amplitudes corresponding to a single tap. Thus the CIR vector is given by

$$\mathbf{h} = [h_0, h_1, \ldots, h_{P-1}]^T,$$

where P is the number of significant taps in CIR. The channel coefficients, h_l, are assumed to be distributed as $h_l \sim \mathcal{CN}(0,\sigma_l^2)$, $\Re\{h_l\} \sim \mathcal{N}(0,\sigma_l^2/2)$, and $\Im\{h_l\} \sim \mathcal{N}(0,\sigma_l^2/2)$. Therefore, the samples of the time-domain channel, $\mathbf{h}$, is circular Gaussian distributed as $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \varSigma)$, where $\varSigma$ of $\mathbf{h}$, the diagonal covariance matrix, is assumed to be known [16]. Considering the AWGN at the receiver, $r(n)$ in (6) is given by

$$r(n) = \exp(j2\pi\Delta f_c nT)\sum_{l=0}^{P-1} h_l x(n-l) + w(n), \tag{7}$$

where $w(n)$ is the independent circular symmetric additive Gaussian noise at the receive antenna with mean zero and variance σ_w^2. Thus $r(n)$ in (7) can be expressed as,

$$r(n) = \exp(j2\pi\varepsilon n/M)\sum_{l=0}^{P-1} h_l x(n-l) + w(n). \tag{8}$$

[2] Notations: $\hat{\mathbf{B}}$ denotes the estimate of $\mathbf{B}$ and $\bar{\mathbf{B}}$ denotes the actual value of $\mathbf{B}$. $\hat{\boldsymbol{\gamma}}(\mathbf{r})$ denotes an estimator of $\boldsymbol{\gamma}$, where $\mathbf{r}$ is the observation vector. $\Im(\mathbf{B})$ and $\Re(\mathbf{B})$ denote the imaginary and real parts of $\mathbf{B}$, respectively. $\tilde{\mathbf{h}}$ and $\mathbf{h}$ denote time-domain and frequency-domain signals, respectively. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. $\mathbf{B}^T$, $\mathbf{B}^*$, and $\mathbf{B}^H$ denote transpose, complex conjugate, and Hermitian of $\mathbf{B}$, respectively. $[\mathbf{B}]_{a,b}$ denotes the (a, b)th element of $\mathbf{B}$. $\circ$ and $\otimes$ represent Hadamard product and Kronecker products, respectively and $\|\mathbf{y}\|_n$ denotes l_n-norm of $\mathbf{y}$. $diag[\mathbf{y}]$ represents a diagonal matrix with $\mathbf{y}$ as diagonal and $diag[\mathbf{B}]$ denotes the diagonal of $\mathbf{B}$ as a column vector. Det($\mathbf{B}$) and Tr($\mathbf{B}$) represent determinant and trace of $\mathbf{B}$, respectively. $\frac{\partial(\mathbf{B})}{\partial\eta}$ represents the partial derivative of $\mathbf{B}$. $p(.)$ denotes pdf and $\mathbb{E}_{ab}[.]$ denotes the expectation over a and b. $\varDelta$ represent partial derivative operator, where $\varDelta_{\gamma} = [\frac{\partial}{\partial\gamma_1}, \frac{\partial}{\partial\gamma_2}, \ldots]^T$, and $\varDelta_{\gamma}^{\alpha} = \varDelta_{\gamma}\varDelta_{\alpha}^T$.