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The classroom assignment problem: Complexity, size reduction and heuristics

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ABSTRACT

In this paper, we investigate a compound of the exam timetabling problems which consists of assigning a set of independent exams to a certain number of classrooms. We can define the exam timetabling problem as the scheduling of exams to time slots in first stage and at a second stage, the assignment of a set of exams extracted from one time slot to some available classrooms.

Even though the formulation of this problem looks simple as it contains only two sets of constraints including only binary variables, we show that it belongs to the class of NP hard problems by reduction from the Numerical Matching with Target Sum problems (NMTS).

In order to reduce the size of this problem and make it efficiently solvable either by exact method or heuristic approaches, a theorem is rigorously demonstrated and a reduction procedure inspired from the dominance criterion is developed. The two methods contribute in the search for a feasible solution by reducing the size of the original problem without affecting the feasibility. Since the reduction procedures do not usually assign all exams to classrooms, we propose a Variable Neighbourhood Search (VNS) algorithm in order to obtain a good quality complete solution. The objective of VNS algorithm is to reduce the total classroom capacity assigned to exams. A numerical result concerning the exam of the main session of the first semester of the academic year 2009–2010 of the Faculty of Economics and Management Sciences of Sfax shows the good performance of our approach compared with lower bound defined as the sum of the total capacity of all assigned classrooms and the total size of the remaining exams after reduction.

1. Introduction

The exam timetabling problem is a part of the educational timetabling problem which includes also the course timetabling in universities or in high schools. This problem is encountered by any educational institution at least twice per academic year and needs efforts and times to be performed. This is the reason that researchers focus their effort in discovering new algorithms and methods to solve this difficult problem and automate its generation [7,8]. According to Qu et al. [36], "exam timetabling problems can be defined as assigning a set of exams into a limited number of time slots (time periods) and rooms (of certain capacity), subject to a set of constraints". This problem is usually solved in two phases: exam-time slot assignment phase and exam-classroom assignment phase. Most researchers have investigated the first phase by several optimization techniques coming from artificial intelligence to operations research. The classroom assignment problem focuses on the assignment of classrooms to exams that have to be taken by independent students of the institution during any time slot. This problem has to be solved in order to find a final solution to the exam timetabling in any academic institution. There is no abundant literature on this part of the exam timetabling problem which makes it an object of research topic in operations research and artificial intelligence. By reviewing the literature, we encounter several techniques that attempt to find a better solution to exam timetabling or to automate its generation. The assignment of event to venues or classrooms concerns in most studies lectures or courses and not exams. Our main objective is to find a better solution for the real world problem of the Faculty of Economics and Management Sciences of Sfax (FEMSS) that could be evaluated by minimizing a well defined evaluation function. Furthermore, our approach could be applied to any other educational institution which has to encounter this type of problem.

Among the papers that tackle this problem, we can cite: [1,3,9,10,12,17,18,20,38]. These papers solved the problem of assigning classes to classrooms for lectures or tutorials. The exam timetabling problem in its broader scope has attracted the attention of several researchers and a number of heuristic procedures and

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algorithms are constructed to solve it in a relatively best manner (see the International timetabling competition 2007 (ITC07) results at McCollum et al. [27]). From the different methods and techniques which are investigated in this field we cite: graph colouring [5,28,29], constraint based reasoning [1,4,14], genetic algorithms [34], tabu search [15,39,40], hyperheurisitcs [33] and multiobjective algorithms [31]. The reader is referred to Qu et al. [36] for an extensive recent literature review concerning examination timetabling.

Qu et al. [35] developed an iterative approach that hybridizes graph heuristics adaptively. In fact, an automatic heuristic construction approach is formulated in which largest weighted degree is adaptively hybridized with saturation degree in every stage of the solution construction for the two problems exam timetabling and graph colouring.

Al-Yakoob et al. [2] formulated the exam timetabling problem (ETP) related to a Kuwait university as a mathematical programming model. They distinguish two variants of the problem: first, the exam timetabling problem (ETP) which is specialized on assigning exams to specified time slots and classrooms; second, the proctor assignment problem (PAP) which performs the assignment of proctors' to exams.

Nashat et al. [32] solved the exam timetabling by a revolutionary metaheuristic consisting of scatter search. They combine the scheduling process of assignment of exams to time slots and to classrooms by introducing a compound in the objective function that considers the number of classrooms with violated capacity.

Burke et al. [6] solved the exam timetabling problem by hybrid variable neighbourhood search with a genetic algorithm. Their idea is based on the use of GA to select a subset of neighbourhoods from a large pool for use within VNS framework.

Kahar and Kendall [24] bring to light the importance of the classroom assignment as a second phase for the achievement of the capacitated examination timetabling schedule. They argue that both versions of the examination timetabling problem can be solved by using a two-phase approach.

There are several works in the literature that provide a good solution to the first phase but, to the best of our knowledge, only one paper (Dammak et al. [13]) have been devoted to the second phase (see [24]). In this work, Dammak et al. [13] considered only the constraint satisfaction problem which ensures that all exams are covered with the available classroom capacity and respect the indivisibility of classroom capacity among several exams. The authors proposed a heuristic procedure known as max-size assignment which is based on two prepositions formulated to reduce the size of the problem. The reduction of the size of the problem is performed either in the right or in the left according to arranging the size of the exams and the capacity of the classrooms in non-increasing or in non-decreasing order.

The construction of a feasible solution to the CAP cannot resume all the objectives that have to be achieved according to a managerial point of view. Indeed, there are several objectives to optimize such that: the total used capacity, classroom proximity, number of classrooms assigned, number of invigilators, energy consumed . . .

We can obtain a compromise solution of the major part of these objectives by considering only the first objective. In fact, when we perform the assignment of exams to classrooms by considering the minimization of the total used capacity, this yields an economy in terms of energy, number of invigilators, and number of classroom. The aim of our present study in this paper is to use different techniques for the reduction of the size of the classroom assignment problem (CAP) in order to make it easy to solve. Section 2 gives a formulation of the problem in a form of a binary linear programme with a suitable objective function measuring the total number of seats assigned. Section 3 contains a proof that the CAP is an NP hard in the strong sense by reduction from the Numerical Matching with Target Sums which is known as NP hard problem [19]. Section 4 contains two size reduction procedures: the first one is based on the arrangement in non decreasing order of exams' sizes and classrooms' capacities while the second is based on the dominance criterion. Section 5 applies the VNS metaheuristic to efficiently solve the CAP by using two neighbourhood structures based on insertion and swap moves. Section 6 presents some numerical results based on the real data extracted from the main session of the first semester exam for the academic year 2009–2010 of the Faculty of Economics and Management Science of Sfax. We compare the performance of our procedures with the reference to a well defined lower bound expressing the optimal value that could be obtained when assigning classrooms to exams. Finally, Section 7 concludes the paper and provides new research directions in this field.

2. Problem formulation

In the variable over-sized bin packing problem, we consider m bins with different capacities to be filled with n items having different sizes. The problem is to assign the items to bins so that the content of each bin i exceeds a given size a_i. The problem of bin packing has been extensively studied in the literature taking into account all its variants [11,23].

Concerning our real case, the bins play the role of exams with different sizes a_i, $i = 1,2, \ldots, m$. The items can be considered as the classrooms with capacities b_j, $j = 1, 2, \ldots, n$. Our objective is to assign classrooms to exams in such away that the total capacity of classrooms assigned to an exam has to exceed its size. The formulation of the classroom assignment, as given in [13], can be presented as follows:

$$(P)\begin{cases}\displaystyle\sum_{j=1}^{n} b_j X_{ij} \geq a_i \quad \forall i \in \{1, \ldots, m\} \quad (1)\\ \displaystyle\sum_{i=1}^{m} X_{ij} \leq 1 \quad \forall j \in \{1, \ldots, n\} \quad (2)\end{cases}$$

The set of constraints (1) indicates for each exam e_i, the total number of seats assigned to this exam has to exceed it size.

The set of constraints (2) indicates that each classroom is assigned to at most one exam. If we need to schedule several exams into one room, we replace the right side of the constraint (2) by the desired number of exams.

The binary variables are defined as follows:

$$X_{ij} = \begin{cases} 1 & \text{if classroom } s_j \text{ is assigned to exam } e_i \\ 0 & \text{otherwise}\end{cases} \quad \forall i \in \{1, \ldots, m\}, \forall j \in \{1, \ldots, n\}$$

E, the set of exams; S, the set of classrooms; m, the number of exams (bins); n, the number of classrooms (items).

This problem can be converted into a binary linear programme by inserting a suitable objective function. The linear programme is defined as follows:

Programme 1: (P_1) minimizing the total used capacity

$$(P_1)\begin{cases}\displaystyle Min\, f(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} b_j X_{ij}\\ \displaystyle\sum_{j=1}^{n} b_j X_{ij} \geq a_i \quad \forall i \in \{1, \ldots, m\}\\ \displaystyle\sum_{i=1}^{m} X_{ij} \leq 1 \quad \forall j \in \{1, \ldots, n\}\end{cases}$$

In this programme, our main objective when we assign classrooms to exams is to minimize the number of idle seats.