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Cooperative spectrum sensing against noise uncertainty using Neyman–Pearson lemma on fuzzy hypothesis test

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ABSTRACT

In this paper, we consider the problem of cooperative spectrum sensing in the presence of the noise power uncertainty. We propose a new spectrum sensing method based on the fuzzy hypothesis test (FHT) that utilizes membership functions as hypotheses for the modeling and analyzing such uncertainty. In particular, we apply the Neyman–Pearson lemma on the FHT and propose a threshold-based local detector at each secondary user (SU) in which the threshold depends on the noise power uncertainty. In the proposed scheme, a centralized manner in the cooperative spectrum sensing is deployed in which each SU sends its one bit decision to a fusion center. The fusion center makes a final decision about the absence/presence of a primary user (PU). The performance of the PU's signal detection is evaluated by the probability of signal detection for a specific signal to noise ratio when the probability of false alarm is set to a fixed value. The performance of the proposed algorithm is compared numerically with two classical threshold-based energy detectors. Simulation results show that the proposed algorithm considerably outperforms the methods with a bi-thresholds energy detector and a simple energy detector in the presence of the noise power uncertainty.

1. Introduction

In recent years, the spectrum scarcity has been a challenging problem in wireless communication systems. The main reasons are the development of various wireless technologies and an increase in the demand for higher rate wireless services. Moreover, the spectrum allocation policy is such inflexible and inefficient so that many portions of the licensed spectrum are not utilized during significant time periods [1]. One heuristic solution to improve the efficiency of the spectrum utilization is the use of Cognitive Radio (CR) technology [2,3]. In this technology, the unlicensed users known as secondary users (SUs), are allowed to use the licensed spectrum bands in an opportunistic manner. One challenge faced in a CR system is that the SUs must accurately sense the spectrum, realize the spectrum holes for their transmissions and vacate the frequency band as soon as the primary users (PUs) start their transmissions [4,5]. Therefore, the spectrum sensing is an essential task to detect the presence of the PU that can be performed either individually or cooperatively. The decision on the presence or absence of the PU in the non-cooperative spectrum sensing is performed individually by each SU, while in the cooperative case, the spectrum sensing is performed by collaboration between a group of SUs to mitigate some problems of spectrum sensing such as multipath fading, shadowing effects, and hidden PUs [6].

In recent years, many studies have been devoted on the cooperative spectrum sensing and many approaches are provided for detecting the PU signals including the cyclostationary feature detection [7] and the energy detection [8]. In most of the work, the power of the noise is assumed to be previously known for threshold setting. However, due to the limited sensing time and the fluctuation of the noise power, the power of the noise is not known precisely and the performance of some classical cooperative spectrum sensing techniques is susceptible to the noise power uncertainty. The main sources of the noise power uncertainty are the noise uncertainty of the receiver device due to the non-linearity and the thermal noise of the components, and the environment noise uncertainty caused by the transmissions of other users [9,10]. Several techniques have been proposed to mitigate the noise power uncertainty in cooperative-based CR systems [11–13]. The cooperative spectrum sensing method in the presence of the noise uncertainty in [11] is based on an energy detector that uses three thresholds for local sensing, while the proposed cooperative approach in [12], is based on a cooperative covariance and eigenvalue based detection approach. The methods proposed

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in [11] and [12] send local sensing results to a fusion center and hence occupy more bandwidth of the control channel. The authors in [13] utilize a bi-thresholds energy detector in each SU where one bit local decision is sent by the SU to a fusion center.

In this paper, we investigate a centralized hard decision-based cooperative spectrum sensing problem in the presence of the noise power uncertainty using fuzzy set theory concepts as a mathematical framework to model such uncertainty. We propose a new spectrum sensing method based on the fuzzy hypothesis test (FHT) that utilizes membership functions as hypotheses for modeling and analyzing the noise power uncertainty. In particular, we apply the Neyman–Pearson lemma on the FHT and propose a threshold-based local detector at each SU in which the threshold depends on the noise power uncertainty. In the proposed scheme, a centralized manner in the cooperative spectrum sensing is deployed in which each SU sends its one bit decision to a fusion center. The fusion center makes a final decision on the absence/presence of the PU. The performance of the PU's signal detection is evaluated by the probability of the signal detection for a specific Signal to Noise Ratio (SNR) when the probability of the false alarm is set to a fixed value. We compare numerically the performance of the proposed algorithm with two classical threshold-based energy detectors. Simulation results show that the proposed algorithm considerably outperforms the methods with a bi-thresholds energy detector and a simple energy detector in the presence of the noise power uncertainty.

The rest of the paper is organized as follows. In Section 2, the FHT background and the system model are provided. The sensing algorithm using the Neyman–Pearson lemma for the FHT is presented in Section 3. Simulation results are presented in Section 4. Finally, conclusions are drawn in Section 5.

Notations: Throughout this paper, we use boldface lower case letters to denote vectors. A Gaussian random variable with mean m and variance σ^2 is represented by $x\sim\mathcal{N}(m,\sigma^2)$. We use $\mathbb{E}[.]$ as the expectation operator, and $\mathbb{P}\{.\}$ for representing the probability of the given event. The signs "$\lesssim$" and "$\gtrsim$" mean "almost smaller" and "almost greater", respectively, and $Q(.)$ is the complementary cumulative distribution function, which calculates the tail probability of a zero mean unit variance Gaussian variable, i.e. $Q(x)=\int_x^\infty \frac{1}{\sqrt{2\pi}}\exp(-t^2/2)dt$. Also, $(.)^T$ stands for the transpose of a matrix or vector.

2. FHT background and system model

2.1. Fuzzy hypothesis test

The conventional binary detection problem is a decision between the two crisp hypotheses H_0, known as the null hypothesis versus the alternative hypothesis H_1. In some situations, we face many practical problems in which the observed data are associated with some uncertainty. Over the past years, there have been some efforts to analyze this uncertainty using the fuzzy set theory [14–16]. Taking the uncertainty into account in the hypothesis test introduces an interesting problem called Fuzzy Hypothesis Test (FHT). To introduce the FHT, suppose that we are interested to test the value of the mean parameter of a normal distribution denoted by θ based on the observations. In the ordinary case, we use the test $H_0:\theta=\theta_0$ against $H_1:\theta\neq\theta_0$. Due to the uncertainty on the model parameter, a more realistic hypothesis can be written as

$$\begin{cases} H_0: & \theta \text{ is close to } \theta_0 \\ H_1: & \theta \text{ is away from } \theta_0. \end{cases} \tag{1}$$

Such hypothesis test is a FHT which has been extensively investigated in the literature [17–21].

Definition 2.1. Any hypothesis of the form "$H:\theta$ is $H(\theta)$" is called a fuzzy hypothesis, implying that θ is in a fuzzy set of Θ with the membership function $H(\theta)$ which is a function from Θ to [0, 1] [20].

Note that the crisp hypothesis "$H_i:\theta\in\Theta_i$" is a fuzzy hypothesis with the membership function "$H_i\left(\theta\right)=1$ at $\theta\in\Theta_i$, and zero otherwise".

2.2. Neyman–Pearson lemma for FHT

Let $\mathbf{X}=(X_1,\ldots,X_n)^T$ be a random sample vector, with the observed value $\mathbf{x}=(x_1,\ldots,x_n)^T$ where X_i has the probability density function (pdf) $f(x_i;\theta)$ with the unknown parameter $\theta\in\Theta$, in which Θ is the parameter space. Suppose the two membership functions $H_0(\theta)$ and $H_1(\theta)$ are known and we would like to test

$$\begin{cases} H_0: & \theta \text{ is } H_0(\theta), \\ H_1: & \theta \text{ is } H_1(\theta). \end{cases} \tag{2}$$

The aim is to accept or reject H_0 on the basis of $\mathbf{x}$. In other words, we would like to make a test function $\phi(\mathbf{X})$ such that $\phi(\mathbf{x})$ is the probability of rejecting H_0 if $\mathbf{X}=\mathbf{x}$ is observed. We use the above FHT for our analysis in the subsequent section.

Definition 2.2. For the fuzzy hypothesis test in (2.1) and the test function $\phi(\mathbf{X})$, the false alarm probability denoted by P_{fa} is defined by [22]

$$P_{fa}=\frac{1}{M}\int_\theta H_0(\theta)\mathbb{E}_\theta\left[\phi\left(\mathbf{X}\right)\right]d\theta, \tag{3}$$

where

$$M=\int_\theta H_0(\theta)d\theta, \tag{4}$$

and

$$\mathbb{E}_\theta[\phi(\mathbf{X})]=\int \phi(\mathbf{x})f(\mathbf{x};\theta)d\mathbf{x}, \tag{5}$$

where $f(\mathbf{x};\theta)$ is the joint pdf of $\mathbf{X}$ with the unknown parameter $\theta\in\Theta$.

In the next theorem, we present the Neyman–Pearson lemma for the above fuzzy hypothesis test.

Theorem 2.3. *Let $\mathbf{X}=(X_1,\ldots,X_n)^T$ be a random sample vector, with the observed value $\mathbf{x}=(x_1,\ldots,x_n)^T$ where X_i has the pdf $f(x_i;\theta)$ with the unknown parameter $\theta\in\Theta$. Defining $\Lambda(\mathbf{x})=(\int_\theta f(\mathbf{x};\theta)H_1(\theta)d\theta)/(\int_\theta f(\mathbf{x};\theta)H_0(\theta)d\theta)$, for the fuzzy hypothesis test problem in (2.1)*

(a) *Any test with the test function*

$$\phi(\mathbf{x})=\begin{cases} 1 & \Lambda(\mathbf{x})>\kappa, \\ \delta(\mathbf{x}) & \Lambda(\mathbf{x})=\kappa, \\ 0 & \Lambda(\mathbf{x})<\kappa, \end{cases} \tag{6}$$

for some $\kappa\geq 0$ and $0\leq\delta(\mathbf{x})\leq 1$, is the best test of size P_{fa}.

(b) *For $0\leq\alpha\leq 1$, there exists a test of form (6) with $\delta(\mathbf{x})=\delta$ (a constant), for which $P_{fa}=\alpha$.*

See [22] for the proof.

2.3. System model

Suppose in a homogeneous cognitive radio network all the SUs use the same protocol for the local spectrum sensing. The network contains K secondary users indexed by $\mathcal{K}=\{1,\ldots,K\}$ to exploit the spectrum holes. We use a centralized manner in the cooperative