COMPUTER METHODS AND PROGRAMS IN BIOMEDICINE 136 (2016) 55–64

journal homepage: www.intl.elsevierhealth.com/journals/cmpb

Traumatic brain injury in pedestrian–vehicle collisions: Convexity and suitability of some functionals used as injury metrics

D. Sánchez-Molina [a,*], C. Arregui-Dalmases [b,c], J. Velázquez-Ameijide [b], M. Angelini [a], J. Kerrigan [d], J. Crandall [d]

[a] UPC-RMEE-EEBE, Barcelona, Spain
[b] UPC-EM-EEBE, Barcelona, Spain
[c] CAB, Charlottesville, VA, USA
[d] UVA-CAB, Charlottesville, VA, USA

ARTICLE INFO

Article history:
Received 15 February 2016
Received in revised form
19 July 2016
Accepted 12 August 2016

Keywords:
Traffic collision
Traumatic brain injury
Injury metrics
HIC
HIP
BRIC

ABSTRACT

Background and objective: Abrupt accelerations or decelerations can cause large strain in brain tissues and, consequently, different forms of Traumatic Brain Injury (TBI). In order to predict the effect of the accelerations on the soft tissues of the brain, many different *injury metrics* have been proposed (typically, an injury metric is a real valued functional of the accelerations). The objective of this article is to make a formal and empirical comparison, in order to identify general criteria for reasonable injury metrics, and propose a general guideline to avoid ill-proposed injury metrics.
Methods: A medium-sized sample of vehicle–pedestrian collisions, from Post Mortem Human Subject (PMHS) tests, is analyzed. A statistical study has been conducted in order to determine the discriminant power of the usual metrics. We use Principal Component Analysis to reduce dimensionality and to check consistency among the different metrics. In addition, this article compares the mathematical properties of some of these functionals, trying to identify the desirable properties that any of those functionals needs to fulfill in order to be useful for optimization.
Results: We have found a pair-wise consistency of all the currently used metrics (any two injury metrics are always positively related). In addition, we observed that two independent principal factors explain about 72.5% of the observed variance among all collision tests. This is remarkable because it indicates that despite high number of different injury metrics, a reduced number of variables can explain the results of all these metrics. With regard to the formal properties, we found that essentially all injury mechanisms can be accounted by means of scalable, differentiable and convex functionals (we propose to call *minimization suitable injury metric* any metric having these three formal properties). In addition three useful functionals, usable as injury metrics, are identified on the basis of the empirical comparisons.
Conclusions: The commonly used metrics are highly consistent, but also highly redundant. Formal minimal conditions of a reasonable injury metric has been identified. Future proposals of injury metrics can benefit from the results of this study.

* *Corresponding author.* UPC, RMEE, COMTE D'URGELL, 187, 08036 Barcelona, Spain.
E-mail address: david.sanchez-molina@upc.edu (D. Sánchez-Molina).
http://dx.doi.org/10.1016/j.cmpb.2016.08.007

1. Introduction

Traumatic brain injury (TBI) is a major global health problem. Country-based estimate of incidences ranges from 108 to 332 new cases admitted to the hospital per 100,000 population per year [1]. On average, 39% of patients with severe traumatic brain injury die from their injury [2]. On the other hand, the design of restraint systems has had an impact on the number and type of injuries in traffic collisions. Currently, the design of restraint systems is assessed using some injury metrics. Indeed, a large number of different injury metrics have been proposed for different purposes [3].

This study presents a theoretical overview of *Injury Metrics* and considers what kind of mathematical properties are desirable for such a metric to be suitable for damage minimization and the optimization of restrain systems. The existing metrics are systematically considered from a formal point of view and its mathematical properties are explored. Finally, a comparison of the prediction of different metrics is made using a medium-sized sample of vehicle–pedestrian collision with Post Mortem Human Subjects (PMHS). Sections 2 and 3 provide a mathematical overview and proper definitions of the commonly used Injury Metrics for TBI. In Section 4, the empirical predictions are presented and three new Injury Metrics are introduced. The new metrics are suggested by physical arguments and by the results obtained. Some discussion of the results is provided in Section 5. Most of the mathematical details are provided in the final Appendix.

2. Injury metrics

2.1. General description

An injury metric is a real valued function of the "acceleration curve" $(\mathbf{a}(t), \boldsymbol{\alpha}(t))$, where $\mathbf{a}(t)$ represents the linear acceleration of the center of mass of the head and $\boldsymbol{\alpha}(t)$ the rotational acceleration of the skull. In order to properly define an injury metric we need to specify the domain of definition for this injury metric. Being the arguments $\mathbf{a}(t)$ and $\boldsymbol{\alpha}(t)$, we consider first the vector space of all possible linear and rotational accelerations satisfying some regularity conditions. Mathematically, it is convenient for each component of the acceleration to be integrable over time. For these reasons, we consider the *Hilbert vector space* of [equivalence classes of] square-integrable functions $L^2(\mathbb{R})$ for each component. A function $f(t) \in L^2(\mathbb{R})$ satisfies:

$$\int_{\mathbb{R}} |f(t)|^2 \, dt < \infty \tag{1}$$

Thus for the linear accelerations we consider the Hilbert space [given by the Cartesian product $\mathbf{L}^2(\mathbb{R}) = L^2(\mathbb{R}) \times L^2(\mathbb{R}) \times L^2(\mathbb{R})$] and similarly for the rotational accelerations. The squared value in Eq. (1) is needed in order to ensure that we can define an abstract inner product in the space of accelerations (in practice, this technical mathematical condition is not a restriction because accelerations are different from zero only during a finite time interval).

A typical injury metric can be represented by a functional, defined on a [convex] set of the Hilbert space $\mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R})$. Typically this type of functional involves computing integrals, taking maxima or particular values of the acceleration curves $(\mathbf{a}(t), \boldsymbol{\alpha}(t)) \in \mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R})$. We can ask for the reasonable mathematical properties of an injury metric to be useful (continuity, existence of optimal curves, differentiability, convexity, existence of minima, etc.). In particular we are interested in comparing different processes of the impact of a human head against the structure of a vehicle or an abrupt deceleration of the head. In order to compare severity, we are particularly interested in curves that imply a complete deceleration after a distance d in the direction of the initial velocity $\mathbf{v}_0$. This distance is given by:

$$\begin{aligned} d - \|\mathbf{v}_0\| T &= \int_0^T \int_0^\tau \hat{\mathbf{u}} \cdot \mathbf{a}(\bar{\tau}) \, d\bar{\tau} \\ &= \int_0^T (T-\tau) \hat{\mathbf{u}} \cdot \mathbf{a}(\tau) \, d\tau = \langle (T-\tau), \hat{\mathbf{u}} \cdot \mathbf{a}(\tau) \rangle \end{aligned} \tag{2}$$

where the versor $\hat{\mathbf{u}} = \mathbf{v}_0 / \|\mathbf{v}_0\|$ is aligned with the initial velocity $\mathbf{v}_0$ and $\mathbf{a}$ represents the linear acceleration (which is different from zero only in the time interval [0, T]). Notice that the second member can be expressed in terms of the inner product $\langle \cdot, \cdot \rangle$ of $L^2(\mathbb{R})$. For this reason we consider the convex set of $\mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R})$ given by:

$$V_{d,\mathbf{v}_0} = \{(\mathbf{a}, \boldsymbol{\alpha}) \in \mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R}) | \langle T - t, \mathbf{a}(t) \cdot \hat{\mathbf{u}} \rangle \le d - \|\mathbf{v}_0\| T\} \tag{3}$$

$V_{d,\mathbf{v}_0}$ is a half-space of $\mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R})$ and, therefore, it is convex (indeed, a half-space is always convex). The requirement for the dominion of comparison to be convex is a crucial technical condition for some comparison of metrics.

2.2. Desirable properties for injury metrics

An injury metric functional $\mathrm{Inj}: V_{d,\mathbf{v}_0} \subset \mathbf{L}^2(\mathbb{R}) \to \mathbb{R}$ is *scalable* if for any $\lambda > 1$, and $\mathbf{a} \in \mathbf{L}^2(\mathbb{R}) := L^2(\mathbb{R}) \times L^2(\mathbb{R}) \times L^2(\mathbb{R})$, we have

$$\mathrm{Inj}(\mathbf{a}) \le \mathrm{Inj}(\lambda \mathbf{a}) \tag{4}$$

This condition ensures that "all else being equal, injury does not decrease if the acceleration increases for each time t". Another convenient condition is *continuity* [or *differentiability*]; this additional condition implies that small changes in the acceleration imply small changes in the effect of the brain tissues. Finally we introduce the notion of convexity related to the existence of minima and/or optimal curves. An injury metric $\mathrm{Inj}(\cdot)$ is *convex* if it is defined on [a convex subset of] $\mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R})$ and if for any $0 \le \mu \le 1$, we have

$$\mathrm{Inj}(\mu \mathbf{a}_1 + (1-\mu)\mathbf{a}_2) \le \mu \mathrm{Inj}(\mathbf{a}_1) + (1-\mu)\mathrm{Inj}(\mathbf{a}_2) \tag{5}$$

This last property is important because it entails the existence of a minimum (if the functional $\mathrm{Inj}(\cdot)$ is strictly convex this minimum is unique) (see Theorem 4 of Appendix for details).

An injury metric is *suitable for minimization (or simply suitable)* if it is scalable, continuous and convex. In fact, we will see in the next section that many of the commonly used injury metrics are suitable. This suggests that it is mathematically