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Interacting waves of Davey–Stewartson III system

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ABSTRACT

Modulational unstable regions of the Davey–Stewartson (DS) III system have been determined from the generalized dispersion relation associating the frequency and wavenumber of the modulating perturbations. By means of the multilinear variable separation approach, the variable separation solution for the DS III equation is obtained with two arbitrary functions of (x, t) and two arbitrary functions of (y, t), which can be utilized to generate various $(2+1)$-dimensional localized excitations. Particular attention is paid on the interacting waves between periodic multivalued foldons and single-valued dromions, which can be viewed as periodic extensions of single foldon–dromion excitations.

1. Introduction

Generally, it is difficult to obtain exact solutions of nonlinear partial differential equations (NPDEs) which stem from miscellaneous nonlinear phenomena in various fields such as Physics [1], Biology [2], Economy [3], Meteorology [4], and so on. To date, many effective methods of Mathematical Physics have been established to obtain exact solutions of NPDEs, especially for the so-called integrable systems or soliton equations. For instance, the inverse scattering transformation method, as a nonlinear generalization of the linear Fourier transformation method, can be used to solve NPDEs with different boundary conditions and initial conditions [5]. Many masterly relations have been constructed to connect solutions of two same or different systems, which could eventually generate new solutions from known ones via the methods such as the Bäcklund transformation [6], Darboux transformation [7], nonlinear superposition [8], symmetry approach [9], and so on.

It is known that equations with multivalued solutions are encountered in different fields and often play an important role there, and thus the problem of determining general classes of solutions of equations with multivalued solutions becomes an important factor for understanding the general structure of the set of solutions of nonlinear hyperbolic equations widely used in practical problems [10]. A new class of exact multivalued real solutions of several multidimensional second-order nonlinear equations including the equations for the propagation of electromagnetic waves in arbitrary polarized dielectrics and of acoustic waves in gases was obtained [10]. Recently, multivalued solitons have been investigated in many nonlinear systems, such as the fluids, liquid crystals, optics, Bose–Einstein condensates, and so on [11–14]. Therefore, it is interesting and meaningful to study interactions involving multivalued solutions. For instance, different kinds of interacting features between the localized multivalued loop solitons within a ferrite slab have been extensively investigated analytically and numerically in [15]. Two dimensional localized structures of multivalued solitons, called folded solitary waves and foldons, have been discovered [16].

The multilinear variable separation approach (MLVSA), as a nonlinear extension of the separation of variables for linear differential equations, has been proposed to search for solutions with arbitrary low-dimensional functions. Many nonlinear

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differential equations have been solved, for instance, the $(1+1)$-dimensional equations such as the coupled integrable dispersionless system [17], the $(2+1)$-dimensional equations such as the Broer–Kaup–Kupershmidt system [18], the $(3+1)$-dimensional equations such as the Jimbo–Miwa equation [19], and even differential difference equations such as the special Toda equation [20]. As a result, a rather universal formula is found for some quantities or potential fields, which involves arbitrary low-dimensional functions for integrable systems, and thus a sense of integrability, solvability by the multilinear variable separation approach, can be explained [21].

In this paper, the MLVSA is successfully applied to the Davey–Stewartson (DS) III system to generate multilinear variable separation solutions with arbitrary functions, from which interacting waves involving multivalued foldons are studied. The DS III system

$$iQ_t + \sigma_3(Q_{yy} - Q_{xx}) + [A, Q] = 0, \tag{1}$$

$$\begin{pmatrix} A_{1x} & 0 \\ 0 & A_{2y} \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} (Q^2)_y & 0 \\ 0 & (Q^2)_x \end{pmatrix}, \tag{2}$$

comes from the consistency condition $[T_1, T_2]\psi = 0$ of two linear problems

$$T_1\psi = \left\{2\begin{pmatrix} \partial_x & 0 \\ 0 & \partial_y \end{pmatrix} + Q\right\}\psi = 0, \tag{3}$$

$$T_2\psi = \frac{1}{2}\left\{i\partial_t + \partial_x^2 + \partial_y^2 + \begin{pmatrix} 0 & q_x \\ r_y & 0 \end{pmatrix} + A\right\}\psi = 0, \tag{4}$$

where

$$A = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}, \qquad Q = \begin{pmatrix} 0 & q \\ r & 0 \end{pmatrix}. \tag{5}$$

The explicit form of the DS III system can be rewritten as

$$ir_t + r_{xx} - r_{yy} + r(A_2 - A_1) = 0, \tag{6}$$

$$iq_t + q_{yy} - q_{xx} + q(A_1 - A_2) = 0, \tag{7}$$

$$A_{1x} = -\frac{1}{2}(qr)_y, \qquad A_{2y} = -\frac{1}{2}(qr)_x. \tag{8}$$

It is known that the $(2+1)$-dimensional DS equation is an isotropic Lax integrable extension of the well known $(1+1)$-dimensional nonlinear Schrödinger (NLS) equation. Eqs. (6)–(8) are a version of the DS equation for two complex-valued functions q and r. In the case of $r = \pm q^*$ (* denotes the complex conjugate), it reduces to the DS I and DS II equations. Generally, this system can describe the propagation of two-dimensional nonlinear waves in dispersive media with cubic nonlinearity and quadratic dispersion. Generalized Bäcklund gauge transformations depending on arbitrary functions were introduced and wave solutions were generated [22]. In the $(2+1)$-dimensional case, it is proved that the gauge equivalence also takes place between the DS equation and the $(2+1)$-dimensional continuous Heisenberg ferromagnet model [23]. The N-fold Bäcklund transformation for the DS III system has been constructed to yield the N-soliton solution in a single step [24].

The paper is organized in the following fashion. In Section 2, a nonlinear dispersion relation is derived for Eqs. (6)–(8) dealing with the modulational instabilities of a constant amplitude carrier wave. In Section 3, the MLVSA is applied to Eqs. (6)–(8), and the multilinear variable separation solution is obtained including two arbitrary functions of (x, t) and two arbitrary functions of (y, t). Interacting waves can thus be naturally constructed and studied. In Section 4, special wave interactions between periodic foldons and dromions are graphically displayed. The final section is devoted to summary and discussion.

2. Modulational instability analysis

In order to carry out the modulational instability analysis on the system of Eqs. (6)–(8), we first have to find its equilibrium state which is usually a simple and exact monochromatic wave solution. In our case, an equilibrium state can be obtained by assuming

$$q = q_0 e^{i\omega_1 t}, \qquad r = r_0 e^{i\omega_2 t}, \qquad A_1 = A_{10}, \qquad A_2 = A_{20}, \tag{9}$$

where the constants $\omega_1,\ \omega_2,\ A_{10},\ A_{20}$ are real and $q_0,\ r_0$ can be complex. The substitution of Eq. (9) into Eqs. (6)–(8) leads to

$$\omega_1 = -\omega_2 = A_{10} - A_{20}. \tag{10}$$

Thereafter, a small perturbation is added on the above equilibrium state and thus the linearized (take the first-order terms of ϵ) DS III system can be obtained by inserting

$$q = (q_0 + \epsilon Q)e^{i(A_{10}-A_{20})t}, \qquad r = (r_0 + \epsilon R)e^{i(A_{20}-A_{10})t}, \tag{11}$$

$$A_1 = A_{10} + \epsilon U, \qquad A_2 = A_{20} + \epsilon V, \tag{12}$$