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Asymptotic stability of a viscoelastic problem with Balakrishnan–Taylor damping and time-varying delay

Jum-Ran Kang [a], Mi Jin Lee [b], Sun Hye Park [c,*]

[a] *Department of Mathematics, Dong-A University, Busan 604-714, South Korea*
[b] *Department of Mathematics, Pusan National University, Busan 609-735, South Korea*
[c] *Center for Education Accreditation, Pusan National University, Busan 609-735, South Korea*

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ABSTRACT

A viscoelastic problem with Balakrishnan–Taylor damping and time-varying delay of the form

$$u_{tt} - (a + b\|\nabla u\|^2 + \sigma(\nabla u, \nabla u_t))\Delta u + \int_0^t g(t-s)\Delta u(s)ds + \mu_1 f_1(u_t(x,t)) + \mu_2 f_2(u_t(x, t-\tau(t))) = 0$$

is considered. We prove a general stability result for the equation without the condition $\mu_2 > 0$ by establishing some Lyapunov functionals which are equivalent to the energy of the equation instead of multiplier technique and using some properties of convex functions.

1. Introduction

We investigate a decay result of the energy for a viscoelastic problem with Balakrishnan–Taylor damping and time-varying delay

$$u_{tt} - (a + b\|\nabla u\|^2 + \sigma(\nabla u, \nabla u_t))\Delta u + \int_0^t g(t-s)\Delta u(s)ds$$
$$+\mu_1 f_1(u_t(x,t)) + \mu_2 f_2(u_t(x, t-\tau(t))) = 0 \quad \text{in } \Omega \times (0,\infty), \tag{1.1}$$
$$u = 0 \quad \text{on } \partial\Omega \times (0,\infty), \tag{1.2}$$
$$u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x) \quad \text{in } \Omega, \tag{1.3}$$
$$u_t(x,t) = f_0(x,t) \quad \text{in } \Omega \times [-\tau(0), 0), \tag{1.4}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$, a, b, σ are positive constants, $\mu_1 > 0, \mu_2 \neq 0$ is a real number, $\tau(t) > 0$ represents time-varying delay, and g, f_1, f_2 are some functions which will be specified later. Problem (1.1)–(1.4) is related to the flutter panel equation and the spillover problem with memory and time-varying delay control.

Balakrishnan and Taylor [1] and Bass and Zes [2] introduced Balakrishnan–Taylor damping which arises from a wind tunnel experiment at supersonic speeds. Later, some authors have discussed results on existence and asymptotic behavior of a class of equations with Balakrishnan–Taylor damping (see [3–7] and references therein).

* Corresponding author.
E-mail addresses: pointegg@hanmail.net (J. Kang), jin0624@pusan.ac.kr (M.J. Lee), sh-park@pusan.ac.kr (S.H. Park).

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When $\mu_1 = 0$ and $\mu_2 = 0$ in (1.1), several authors have studied existence of the solutions and stability of the corresponding energy. For examples, Tatar and Zarai [5,7] showed exponential/polynomial decay results under the classical condition of g. Wu [8] proved general decay rates for the problem with nonlinear boundary/interior sources under weakened conditions of g than those of [5,7]. Recently, Park [4] established arbitrary decay rates without imposing the usual relations between g and g' as above.

In this paper, we investigate a general decay result for a viscoelastic problem with time-varying delay appearing in the control term in (1.1). Introducing the time delay term $\mu_2 f_2(u_t(x, t-\tau(t)))$ makes the problem different from those of existing literature. Time delay arises in many applications depending not only on the present state but also on some past occurrences. It may turn a well-behaved system into a wild one, that is, the presence of delay may be a source of instability (see e.g. [9,10]). Thus, the control of partial differential equations with time delay effects has become an active area of research. For examples, Nicaise and Pignotti [10] examined a wave equation with constant delay and they improved the results of [10] by considering time-varying delay instead of constant delay. After that, systems with time-varying delay have been studied by many authors (see [11,12] and references therein). Moreover, Benaissa et al. [13] studied a nonlinear wave equation with time-varying delay of the form

$$u_{tt}(x,t) - \Delta u(x,t) + \mu_1\sigma(t)f_1(u_t(x,t)) + \mu_2\sigma(t)f_2(u_t(x,t-\tau(t))) = 0, \tag{1.5}$$

where $\mu_1, \mu_2 > 0$ and σ, f_1, f_2 satisfy some conditions. They showed a general decay result, which extended previous works, and used the multiplier method and properties of convex functions in order to get desired results.

Regarding the viscoelastic wave equation with constant delay of the form

$$u_{tt}(x,t) - \Delta u(x,t) + \int_0^t g(t-s)\Delta u(x,s)ds + \mu_1 u_t(x,t) + \mu_2 u_t(x,t-\tau) = 0, \tag{1.6}$$

several authors discussed on existence and stability of the solutions under appropriate conditions on μ_1, μ_2, and g (see e.g. [14,15]). For the related problems, we also refer [16–19]. As far as we know, there is not much literature on energy decay for the viscoelastic equation with time-varying delay. Motivated by these results and inspired by the idea of [13], we will investigate general decay rates of energy for problem (1.1)–(1.4) by dropping the restriction $\mu_2 > 0$ and establishing suitable Lyapunov functionals which are equivalent with the corresponding energy. Furthermore, our result improves those of [16].

2. Preliminaries and main results

We use the standard Lebesgue space $L^2(\Omega)$ and Sobolev space $H_0^1(\Omega)$. For a Hilbert space X, we denote $(\cdot,\cdot)_X$ and $\|\cdot\|_X$ the inner product and norm of X, respectively. For simplicity, we denote $(\cdot,\cdot)_{L^2(\Omega)}$ and $\|\cdot\|_{L^2(\Omega)}$ by $(\cdot,\cdot)$ and $\|\cdot\|$, respectively. Let λ be the smallest positive constant such that

$$\lambda\|u\|^2 \le \|\nabla u\|^2 \quad \text{for } u \in H_0^1(\Omega). \tag{2.1}$$

We state the assumptions for the problem (1.1)–(1.4).

(A1) Hypotheses on g.

The kernel function $g : [0,\infty) \to (0,\infty)$ is a bounded C^1 function satisfying

$$a - \int_0^\infty g(s)ds = l > 0 \tag{2.2}$$

and there exists a nonincreasing differentiable function $\zeta : \mathbb{R}_+ \to \mathbb{R}_+$ satisfying

$$g'(t) \le -\zeta(t)g(t) \quad \text{for } t \ge 0. \tag{2.3}$$

(A2) Hypotheses on f_1.

Similarly to [20], $f_1 : \mathbb{R} \to \mathbb{R}$ is a non-decreasing function of the class $C(\mathbb{R})$ such that there exist positive constants $r < 1$, γ_1, γ_2 satisfying

$$\gamma_1|s| \le |f_1(s)| \le \gamma_2|s| \quad \text{for } |s| \ge r. \tag{2.4}$$

Moreover, assume that there exists a convex increasing function $F_1 : \mathbb{R}_+ \to \mathbb{R}_+$ of class $C^1(\mathbb{R}_+) \cap C^2((0,\infty))$ satisfying

$$(i)\, F_1(0) = 0, \tag{2.5}$$

$$(ii)\, F_1 \text{ is linear on } (0,r], \text{ or } F_1'(0) = 0 \text{ and } F_1''(t) > 0 \text{ on } (0,r], \tag{2.6}$$

$$(iii)\, f_1^2(s) \le F_1^{-1}(sh_1(s)) \text{ for } |s| \le r. \tag{2.7}$$

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