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Stability of a reaction–diffusion alcohol model with the impact of tax policy[☆]

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ABSTRACT

To understand the impact of tax policy on the persistence of a drinking behavior, a reaction–diffusion alcohol model with the impact of tax policy is studied, with the focus on the positivity and boundedness of solutions and particularly the asymptotic profile of the equilibria. The basic reproduction number R_0 is calculated, and existence of a drinking-free equilibrium and a unique drinking-present equilibrium is established for R_0 above the threshold value 1. For both of ODE system and reaction–diffusion model, it is shown that the drinking-free equilibrium is globally asymptotically stable if $R_0 \le 1$, and if $R_0 > 1$, we obtain a sufficient condition for global asymptotically stability of the drinking-present equilibrium. Numerical simulations are also provided to illustrate our analytical results.

1. Introduction

Modern medical research shows that drinking wine and beer in right amount is good for one's heath. The scientists in Medical School of Harvard found that drinking wine one or two glasses per day can reduce the incidence of heart disease by 36%. Their research result also shows that wine, beer and liquor have the same effect. However, excessive drinking can cause many diseases. A large number of clinical trials confirmed: Alcohol on the liver injury is the most direct. Alcohol makes the liver cell degeneration and necrosis, it kills a large number of liver cells, and causes transaminase sharp rise. Long-term excessive drinkers are more likely to suffer from alcoholic fatty liver, alcoholic hepatitis or alcoholic cirrhosis. According to a research report of Shanghai Institute of environmental and economic disaster prevention: In the past seven years, alcoholism in patients grew by 28.5 times, the death toll rose by 30.6 times due to drinking the large number of strong liquor. Hence, on the basis of drinking quantity, drinking can be divided into healthy drinking and unhealthy drinking.

From 1990s, many authors begin to discuss the impact of family environment, school environment, social environment, fiscal taxes, laws, regulations on drinking behavior and the impact of alcohol on a number of diseases (see for example [1–5]). Tax policy influence balance of payments and drinking behavior of a region. The higher the taxes, the higher the price of wine is. It leads that many drinkers in the high taxes area cannot afford the high drinking fee. As a result, this part of the people have to choose abstinence. Therefore, the number of drinkers in the high taxes area must be less than the one in the low taxes area. Delcher et al. [6] showed that excise tax increases on beer and spirits were associated with reductions in alcohol-related disease mortality. Modifying tax rates on a single beverage type does not appear to be as effective as doing so on multiple alcoholic beverages simultaneously. The study in Lin et al. [7] provides evidence that alcohol taxation has resulted in an immediate reduction of medical expenditures related to alcohol-attributed diseases. The policy of increasing alcohol

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tax rates may have favorable influences on health care resources related to treating alcohol-attributed diseases. Blecher [8] discussed that raising the tax rates of tobacco, alcohol and sugar can effectively control the problems of smoking, alcohol abuse and obesity. Nelson and McNall [9] get the conclusion as follows: Prior reviews stress taxes as a comprehensive and cost-effective intervention for addressing alcohol-related harms. A review of natural experiments indicates the confidence placed on this measure is too high, and natural experiments in alcohol policy had selective effects on various subpopulations. In addition, Cintrón-Arias et al. [10] discussed a quit drinking model with nonlinear relapse rate, the authors used the method of mathematics to analysis the model, and did a large number of numerical simulation, but they only gave the basic reproduction number, did not discuss the existence of the equilibria and did not give the theoretical proof for the stability of the equilibria. Xiang, Zhu and Huo [11] discussed global dynamics of a quit drinking model with relapse, the paper shown that relapse played an important role in the quit drinking progress. Huo et al. [12–15] also discussed stability, Hopf bifurcation and optimal control strategy of some other smoking, alcohol and viral problems.

Recently, reaction–diffusion equations have been used by many authors in epidemiology as well as virology. Many types of epidemic disease model [16–23]with diffusion have been established by the researchers. In the chapter about migration and population mobility of any geography textbook can be found the following description: Population migration can be divided into internal migration and international migration by space, and the time of the mobility is random. Although the short-term population movement such as travel does not belong to migration, however, the time of such movements are still random. And because of the existence of such a short period of population mobility, the frequency of population movements between different regions of the world is increased, so the randomness of the global population flows are more obviously. Accompanied by population movements, infectious diseases, smoking and alcohol abuse and other personal behavior will spread from one area to other places. On the other hand, we may wish to look at the initial diffusion of alcohol. Dissemination of information on the wine has well documented in the British Museum Brief History of the World [24]. From the initial process of alcohol spread, we can see that they are carried by traders or army to spread. Therefore, the initial diffusion of wine also has many uncertain factors, such as wind direction, weather, traffic and so on, is random. Hence, alcoholism behavior also produced a diffuse with population movements, but so far we have not seen the literature about drinking models with diffusion. Therefore, if we consider the random diffusion in the model, it will be made the model more appropriate to the actual. As we know that Laplace diffusion is described by the random diffusion motion. Hence, if we added the Laplace diffusion factor to the drinking model, that would be an innovative initiative.

Inspired by the above literature, in this paper, we divide the potential drinkers into two parts by different taxes area and divide the drinkers into healthy drinking drinkers and unhealthy drinking drinkers. This classification is practical significance. Some health drinkers may have been to maintain a healthy drinking habits for all their lives, others may develop into alcoholics. Due to the restriction of economic conditions, people are not easy to become alcoholics in the high taxes area. We also consider the total population size N is not a constant. In view of health drinking is good for the human body, so we do not consider the mortality which rely on the drinking in the health drinking compartment. The people who have been subjected to this type of tax policy implications will have a certain percentage of development as the permanent quit drinkers. This assumption is also realistic, and it is very well reflected in the role of tax policy.

The organization of this paper is as follows: In Section 2, the model is given under some assumption and the positive property of the solutions of the model is proved. In Section 3, we obtain the basic reproduction number and find all kinds of equilibria. First, for the model without diffusion, the stability of equilibria is investigated. Then, in Section 4, the stability of reaction–diffusion model are shown. In Section 5, some numerical simulations are given to illustrate our analytical results. The paper ends with a discussion in Section 6.

2. The model formulation

In this section, we consider a reaction–diffusion alcohol model with the impact of different tax policies. Inspired by the literature [16,17,19], we assume that the habitat $\Omega \subset \mathbb{R}^m (m \geq 1)$ is a bounded domain with smooth boundary $\partial\Omega$, and the homogeneous Neumann boundary conditions mean that no population flux crosses the boundary $\partial\Omega$. However, some authors [20–22] gave more specific hypothesis ($\Omega \subset \mathbb{R}^2$ or $\mathbb{R}^3$) about the region Ω on their specific disease model.

2.1. System description

Our model is divided into five compartments, P denotes the potential drinkers in low taxes area, S denotes the potential drinkers in high taxes area, L denotes healthy drinking drinkers, H denotes unhealthy drinking drinkers or problem drinkers, Q denotes permanent abstainers group. The parameters description and transfer diagram as shown in Table 1.

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