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Filling holes with geometric and volumetric constraints

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ABSTRACT

We propose and analyze different methods to reconstruct a function that is defined outside a sub-domain (hole) of a given domain. The reconstructed function is a smooth Powell–Sabin spline that is defined also inside this hole, filling then this lack of information, and, at the same time, fulfills certain global geometric considerations and other local volume constraints on the hole. We give several examples and we include a technique to estimate the volume of the function inside the hole by using just the data of the function where it is known, that is, outside the hole.

1. Introduction

In many situations it is necessary to fill or reconstruct certain function defined in a domain in which there is a lack of information inside one or several sub-domains (holes). Several works related to the field of filling holes have been published in the last few years (e.g., [1–5]). In some practical cases we may have some specific geometrical constraints, of industrial or design type, as the special case of a specified volume inside each one of these holes. In this work we study this particular issue, giving both some theoretical and computational results that assures the feasibility of the corresponding procedures.

The methods and procedures studied in this work manage to find a smooth bivariate spline function that minimizes certain functional, and that properly takes into account all the features that we want this approximate spline to fulfill (the technique of minimizing functionals in order to obtain splines verifying certain required conditions has been used in the literature in the last years, see e.g. [6] and [7]).

Among these possible conditions, there are the volume restrictions, that can be included as an interpolation condition inside the corresponding finite element space constructed, or just as another approximation term to be included in the associated quadratic functional. We can also include some other approximation or interpolation conditions at some specific points outside the holes.

The outline of the paper is as follows: In Section 2 we establish the basic preliminaries and notation related to the triangulations and to the spline functional spaces where we will obtain the filling patches.

In Section 3 we pose the problem of finding a function that fulfills exactly a volume restriction and approximates known function values outside the hole. In Section 4 we find a function that approximates both the known function values and the predetermined volume. In this section we also include a method to estimate an objective volume by using the data outside the hole. In both cases we give some numerical and graphical examples. In both Sections 3 and 4 we obtain the filling functions by a two-steps procedures, whereas in Section 5 we obtain a function that approximates given values outside the hole and

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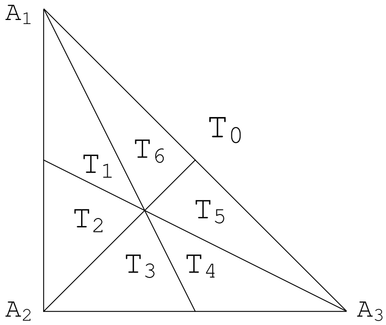

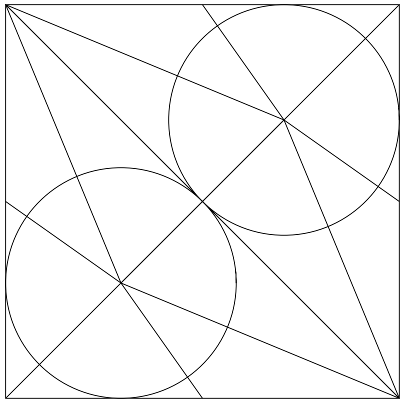

Simple PS sub-triangulation

Fig. 1. PS-triangle T_0.

a predetermined volume in one step. In this section we also outline several one-step procedures that obtain functions that interpolates or approximates known function values and fulfill certain volume restrictions.

Finally, the work closes with some conclusions in Section 6.

2. Preliminaries and notation

In this section we introduce the notation related to the functional spaces we will use throughout the paper and to the triangulations needed to define the spline spaces in which the solutions will be found.

Let $D \subset \mathbb{R}^2$ be a polygonal domain (an open polygonal connected set) and let us consider the Sobolev space $\mathcal{H}^2(D)$, whose elements are (classes of) functions u defined on D such that their partial derivatives (in the distribution sense) $\partial^\beta u$ belong to $\mathcal{L}^2(D)$, with $\beta := (\beta_1, \beta_2) \in \mathbb{N}^2$ and $|\beta| := \beta_1 + \beta_2 \leq 2$. For any open subset X of D we will consider the usual inner semi-products

$$(u, v)_{m,X} := \sum_{|\beta|=m} \iint_X \partial^\beta u \cdot \partial^\beta v, \quad m = 0, 1, 2;$$

the seminorms

$$|u|_{m,X} := (u, u)_{m,X}^{1/2} = \left(\sum_{|\beta|=m} \iint_X (\partial^\beta u)^2 \right)^{1/2}, \quad m = 0, 1, 2;$$

and the norm

$$\|u\|_X = \left(\sum_{m=0}^{2} |u|_{m,X}^2 \right)^{1/2} = \left(\sum_{|\beta|\leq 2} \iint_X (\partial^\beta u)^2 \right)^{1/2}.$$

We will denote $\langle\cdot\rangle_n$ the usual Euclidean norm and $\langle\cdot\ ,\ \cdot\rangle_n$ the Euclidean inner product in $\mathbb{R}^n$.

Given $\alpha \geq 1$, let $\mathcal{T}$ be an α-triangulation of $\overline{D}$, i.e., a triangulation that satisfies the condition

$$1 \leq \frac{R_T}{2r_T} \leq \alpha$$

for all triangles $T \in \mathcal{T}$, R_T and r_T being the radii of the circumscribed and inscribed circles of T, respectively (see e.g. [8]), and let $\mathcal{V}_\mathcal{T}$ be the set of all the vertices of $\mathcal{T}$. We will consider the associated Powell–Sabin triangulation $\mathcal{T}_6$ of $\mathcal{T}$ (see e.g. [9] and Fig. 1), which is obtained by joining the center Ω_T of the inscribed circle of each interior triangle $T \in \mathcal{T}$ to the vertices of T and to the centers $\Omega_{T'}$ of the inscribed circles of the neighboring triangles $T' \in \mathcal{T}$. When T has a side lying on the boundary of D, the point Ω_T is joined to the mid-point of this side, to the vertices of T and to the centers $\Omega_{T'}$ of the inscribed circles of the neighboring triangles $T' \in \mathcal{T}$. Hence, all the micro-triangles inside any $T \in \mathcal{T}$ have the incenter of T as a common vertex.

It is well known [10] that given the values of a function f (defined on $\overline{D}$) and the ones of all its first partial derivatives at all the points of $\mathcal{V}_\mathcal{T}$, there exists a unique S in

$$\mathcal{W} := \mathcal{S}_2^1(D, \mathcal{T}_6) = \left\{ S \in \mathcal{C}^1(\overline{D}) : S|_{T'} \in \mathbb{P}_2(T'), \quad \forall T' \in \mathcal{T}_6 \right\},$$

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