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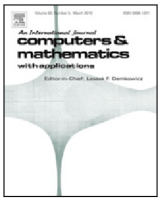

A comparative study of the axisymmetric lattice Boltzmann models under the incompressible limit

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ABSTRACT

A comparative study on four axisymmetric lattice Boltzmann (LB) models, namely, the kinetic theory based model by Guo et al. (2009), the consistent model by Li et al. (2010), the centered scheme model by Zhou (2011), and our model (based on applying the centered scheme to the Guo et al. (2009) model), is conducted both theoretically and numerically. The finite difference interpretation of the LB method by Junk (2001) is applied to evaluate the accuracy of the models under the incompressible limit. Particularly, the finite difference stencils adopted for the spatial gradient terms in the macroscopic axisymmetric Navier–Stokes (N–S) equations are compared. Besides, the numerical performance (i.e., the numerical accuracy, stability and the convergence efficiency) of the models is compared by two benchmark tests, namely, the unsteady-state Womersley flow and the cylindrical cavity flow. The numerical results accord well with the theoretical analysis. Additionally, it is also found that the numerical stability of the axisymmetric LB models is effectively improved by removing the effects from the non-hydrodynamic variables.

1. Introduction

The axisymmetric lattice Boltzmann (LB) models are advantageous in terms of reducing the 3D (three-dimensional) cylindrical flows as well as the coupled heat and mass transfer, which are of practical relevance in engineering practices, to 2D (two-dimensional) cases, thereby bypassing elaborate boundary schemes and saving significant computational costs [1–30]. However, the LB method is inherently for regular lattices within the Cartesian coordinates system, and the macroscopic Navier–Stokes (N–S) equations derived from the standard LB model under the incompressible limit are of the following form

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{1a}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}\left(u_i u_j\right) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right]. \tag{1b}$$

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In contrast, the macroscopic equations for incompressible axisymmetric flows are given as

$$\frac{\partial u_j}{\partial x_j} + \frac{u_r}{r} = 0 \tag{2a}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}\left(u_i u_j\right) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] + \frac{\nu}{r}\left(\frac{\partial u_i}{\partial r} + \frac{\partial u_r}{\partial x_i}\right) - \frac{2\nu u_r}{r^2}\delta_{ir} - \frac{u_i u_r}{r}. \tag{2b}$$

In comparison with Eq. (1), the additional terms in Eq. (2) are attributed as source terms, and thus the first axisymmetric LB model was proposed by Halliday et al. [29] by incorporating the source terms into the standard LB framework. However, it was reported that the recovered macroscopic equations from the Halliday et al. model [29] was incorrect due to a missing term related to the radial velocity component in the momentum equations. Therefore, the Halliday et al. model was corrected and improved by Lee et al. [23] and Reis and Phillips [14,15] to give accurate macroscopic equations. Although the Halliday et al. model and its derivative improved models achieved widespread applications in rotational thermal flows [18,27] and multiphase flows [17,18,31], these predecessor axisymmetric LB models suffered from complex source terms [4], namely, more than ten terms were included in the forcing source terms (accounting for the source terms in Eq. (2b)) of the improved Halliday et al. model, and additional finite difference calculations were involved as in Eq. (50b).

Correspondingly, successive axisymmetric LB models were proposed for simplifying the source terms. The axisymmetric LB model was developed by Chen et al. [32,33] based on the vorticity stream equation, but a Poisson equation had to be solved at each time step and the boundary condition for the vorticity was hard to determine, which makes it inefficient for practical applications. Taking advantage of a centered scheme, Zhou [4] proposed an alternative axisymmetric LB models with the source terms containing fewer terms than the earlier axisymmetric LB models. However, the spatial derivative terms were not avoided in the source terms, and the semi-implicit centered scheme also required information from the neighboring fluid nodes, thus negating improvement in computation efficiency. Subsequently, an innovative way of representing the velocity gradients in the source terms was proposed in Li et al. [2,21] through an additional relaxation term. Essentially, the velocity gradient terms were recovered from the moments of the non-equilibrium part of the distribution function, and thus the finite difference calculations in the source terms were completely avoided. Incorporating this idea, the Zhou model was further revised [3] and then extended to the axisymmetric convection–diffusion equations [22] and axisymmetric compressible flows [9]. In summary, the axisymmetric LB models discussed above were developed under the framework of the standard LB method, and the source terms, i.e., the extra terms in Eq. (2) vis-à-vis Eq. (1), were directly added to the LB equation. Thus, the axisymmetric effects were implemented at the macroscopic level, and such LB models were referred to as top-down models.

Another important category of the axisymmetric LB models is the kinetic theory based models, derived by Guo et al. from the axisymmetric Boltzmann equations [1], and referred to as the bottom-up models. The LB equations were simply the discretized axisymmetric Boltzmann equations, and special distribution function definitions were proposed in the Guo et al. model [1]. Therefore, it was claimed that the axisymmetric conditions were realized at the distribution function level, and the Guo et al. model had the following advantages: (i) the evolution of the radial, axial and azimuthal velocity components were described in a consistent way; and (ii) the source terms were simple and contained no velocity gradients terms, i.e., the velocity gradient source terms were generated in a self-consistent way within the framework of the kinetic theory based axisymmetric LB model. The Guo et al. model also achieved widespread applications spanning thermal flows [5,6], micro-tube flows [34], and multiphase flows [20]. Despite the merits and wide applicability of the Guo et al. model [1], its drawbacks are also clear. The two LB equations in Eq. (3) derived from the axisymmetric Boltzmann equation are coupled together through the equilibrium distribution function definitions in Eqs. (6a) and (7a), and both evolve on the D2Q9 lattice model, which would lead to computational overhead since the evolution of azimuthal velocity distribution function can be resolved on simpler lattice models. Notably, this coupling can be easily removed by applying a revised equilibrium distribution function for the azimuthal velocity, and then the D2Q4 model, which has less discrete particle velocity vectors, is applied [35]. In addition, our recent effort resulted in an alternative kinetic theory based axisymmetric LB model by applying the centered scheme to the Guo et al. model to facilitate the calculations of the macroscopic variables.

In this work, a comparative study is performed on four axisymmetric LB models: the Guo et al. model [1], Li et al. model [2], the revised Zhou model [3] (hereinafter refer to as 'Zhou model') and our model. The source terms of the four models are simpler than the earlier models discussed in the first paragraph, and no additional finite difference calculations are required for the velocity gradient source terms. Moreover, although the semi-implicit centered scheme, applied in the Zhou model and our model, also requires information from the neighboring fluid nodes, the finite difference stencils for the recovered macroscopic equations would not be affected [36]. The present work is targeted at further comparing the different ways of realizing the axisymmetric effects adopted in the four LB models, and then comparing their accuracy, stability and convergence efficiency. First, the finite difference interpretation of the LB method, proposed by Junk [37], is applied to investigate the overall accuracy of the axisymmetric LB models and compare the adopted finite difference stencils for the spatial derivative terms in the macroscopic equations. Then, numerical simulations are performed for validating the theoretical predictions and further comparing the numerical stability and convergence efficiency of the models.

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