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Wronskian and Grammian solutions for a $(2+1)$-dimensional Date–Jimbo–Kashiwara–Miwa equation

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ABSTRACT

In this paper, a $(2+1)$-dimensional Date–Jimbo–Kashiwara–Miwa equation is investigated. Based on the Hirota method and auxiliary variable, Bäcklund transformation is obtained. Under certain conditions, the Hirota bilinear forms can be reduced to the Plücker and Jacobi identities via the Wronskian and Grammian solutions. Through the Wronskian technique and Pfaffian derivative formulae, N-soliton solutions in the Wronskian and Grammian are given. Graphically presented, soliton collisions are elastic both in the Wronskian and Grammian, and after each collision, the solitons keep their amplitudes unchanged except for the phase shifts.

1. Introduction

Nonlinear evolution equations (NLEEs) have been attractive in science and engineering [1–6], and soliton and rational solutions for some NLEEs have been constructed [7–13]. Methods for deriving the soliton solutions for the NLEEs, including the inverse scattering transformation [14], Painlevé technique [15], Bäcklund transformation (BT) [16], Darboux transformation [17–21], Hirota method [22–24], Binary-Bell-polynomial scheme [25–27], first integral method [28], (G'/G) expansion [29,30], Exp-function method [31,32] and homotopy asymptotic [33], have been proposed. Besides, the Wronskian technique has been applied to certain NLEEs such as the Korteweg–de Vries (KdV), modified KdV, Kadomtsev–Petviashvili (KP), Boussinesq and Whitham–Broer–Kaup equations [34–40]. For the high-dimensional NLEEs, there exist the Grammian and Pfaffian solutions [24]. Grammian solutions for the KP equation and Pfaffian solutions for the B-typed KP equation have been presented [41,42].

To study the integrability of the KP hierarchy [43], people have considered the $(3+1)$-dimensional Jimbo–Miwa equation [44]:

$$w_{XXXY} + 3w_{XY}w_X + 3w_Y w_{XX} + 2w_{YT} - 3w_{XZ} = 0, \tag{1}$$

whose bilinear form is the second bilinear equation of the KP hierarchy, i.e.,

$$(D_X^3 D_Y - 3D_X D_T + 2D_Y D_Z)\tau \cdot \tau = 0, \tag{2}$$

via $w = 2(\ln\tau)_X$, with w and τ as the real functions of the variables X, Y, Z and T, D as the Hirota bilinear operator[1] [24]. Eq. (1) has been proved non-integrable [44].

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[1] The Hirota bilinear operators are defined as $D_X^{m_1} D_T^{m_2}(\vartheta \cdot \varsigma) = (\partial_X - \partial_{X'})^{m_1}(\partial_T - \partial_{T'})^{m_2}\vartheta(X,T)\cdot\varsigma(X',T')|_{X'=X,T'=T}$, with ϑ as an analytic function of X and T, ς as an analytic function of the formal variables X' and T', m_1 and m_2 as the non-negative integers [22–24].

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Researchers have also considered the following integrable (2 + 1)-dimensional Date–Jimbo–Kashiwara–Miwa equation which can be extended through Bilinear Form (2) [45,46]:

$$u_{xxxxy} + 4u_{xxy}u_x + 2u_{xxx}u_y + 6u_{xy}u_{xx} + u_{yyy} - 2u_{xxt} = 0, \tag{3}$$

where u is the real function of the variables x, y and t. Via $u = 2(\ln f)_x$, Eq. (3) has been transformed into the following bilinear forms [45]:

$$(D_x^4 + 3D_y^2 + \alpha D_x D_z) f \cdot f = 0, \tag{4a}$$

$$\left(D_x^3 D_y - 3D_x D_t - \frac{\alpha}{2} D_y D_z\right) f \cdot f = 0, \tag{4b}$$

where z is an auxiliary variable, f is the real function of x, y, z and t, α is a constant. Bilinear Forms (4) have been seen to correspond to the first two bilinear equations of the KP hierarchy [43]. Bilinear BT and nonlinear superposition formulae for Eq. (3) have been derived [45]. Lax pair, infinite conservation laws and multi-shock wave solutions for Eq. (3) have been obtained [46].

To our knowledge, the soliton solutions in the Wronskian and Grammian and the related soliton collisions for Eq. (3) have not been reported. In Section 2, based on the exchange formulae [24], we will obtain the bilinear BT for Eq. (3) which is different from that obtained in [46]. In Section 3, N-soliton solutions in the Wronskian for Eq. (3) will be presented, and the soliton propagation and collisions will be shown. In Section 4, we will present the N-soliton solutions in the Grammian for Eq. (3), and show the soliton collisions too. Section 5 will be our conclusions.

2. Bilinear BT for Eq. (3)

Based on Bilinear Forms (4) and assuming that $u' = 2(\ln f')_x$ which satisfies Eq. (3), we consider

$$P_1 = [(D_x^4 + 3D_y^2 + \alpha D_x D_z) f' \cdot f'] f^2 - [(D_x^4 + 3D_y^2 + \alpha D_x D_z) f \cdot f] f'^2, \tag{5a}$$

$$P_2 = \left[\left(D_x^3 D_y - 3D_x D_t - \frac{\alpha}{2} D_y D_z\right) f' \cdot f'\right] f^2 - \left[\left(D_x^3 D_y - 3D_x D_t - \frac{\alpha}{2} D_y D_z\right) f \cdot f\right] f'^2, \tag{5b}$$

where f', u' are both the real functions of x, y, z and t. With the exchange formulae

$$\begin{aligned}
&D_x(D_y f' \cdot f) \cdot (ff') = D_y(D_x f' \cdot f) \cdot (ff'),\\
&D_y(D_x^2 f' \cdot f) \cdot (ff') = D_x[(D_x D_y f' \cdot f) \cdot (ff') - (D_x f' \cdot f)(D_y f \cdot f')],\\
&(D_x D_t f' \cdot f') f^2 - (D_x D_t f \cdot f) f'^2 = 2D_x(D_t f' \cdot f) \cdot (ff'),\\
&(D_x D_z f' \cdot f') f^2 - (D_x D_z f \cdot f) f'^2 = 2D_x(D_z f' \cdot f) \cdot (ff'),\\
&(D_y D_z f' \cdot f') f^2 - (D_y D_z f \cdot f) f'^2 = 2D_y(D_z f' \cdot f) \cdot (ff'),\\
&(D_y^2 f' \cdot f') f^2 - (D_y^2 f \cdot f) f'^2 = 2D_y(D_y f' \cdot f) \cdot (ff'),\\
&(D_x^3 D_y f' \cdot f') f^2 - (D_x^3 D_y f \cdot f) f'^2 = 2D_y(D_x^3 f' \cdot f) \cdot (ff') + 6D_x(D_x D_y f' \cdot f) \cdot (D_x f \cdot f'),\\
&(D_x^4 f' \cdot f') f^2 - (D_x^4 f \cdot f) f'^2 = 2D_x(D_x^3 f' \cdot f) \cdot (ff') + 6D_x(D_x^2 f' \cdot f) \cdot (D_x f \cdot f'),\\
&2D_x(D_x f' \cdot f)(D_x D_y f \cdot f') = D_y(D_x^3 f' \cdot f) \cdot (ff') + D_y(D_x f' \cdot f) \cdot (D_x^2 f \cdot f') + D_x(D_x^2 D_y f' \cdot f) \cdot (ff') + D_x(D_y f' \cdot f) \cdot (D_x^2 f \cdot f'),
\end{aligned}$$

P_1, P_2 can be rewritten as

$$\begin{aligned}
P_1 &= 2D_x(D_x^3 f' \cdot f) \cdot (ff') + 6D_x(D_x^2 f' \cdot f) \cdot (D_x f \cdot f') + 6D_y(D_y f' \cdot f) \cdot (ff') + 2\alpha D_x(D_z f' \cdot f) \cdot (ff')\\
&= 2D_x[(D_x^3 + \alpha D_z - 3D_x D_y) f' \cdot f] \cdot (f \cdot f') + 6D_x[(D_x^2 + D_y) f' \cdot f] \cdot (D_x f \cdot f') + 6D_y[(D_x^2 + D_y) f' \cdot f] \cdot (ff'),
\end{aligned} \tag{6a}$$

$$P_2 = 2D_y(D_x^3 f' \cdot f) \cdot (f \cdot f') + 6D_x(D_x D_y f' \cdot f) \cdot (D_x f \cdot f') - 6D_x(D_t f' \cdot f) \cdot (ff') - \alpha D_y(D_z f' \cdot f) \cdot (ff'). \tag{6b}$$

We spilt Expressions (6), and get the bilinear BT for Eq. (3), i.e.,

$$(D_x^2 + D_y + \lambda) f' \cdot f = 0, \tag{7a}$$

$$(D_x^3 + \alpha D_z - 3D_x D_y - 3\lambda D_x) f' \cdot f = 0, \tag{7b}$$

$$\left(D_t + \frac{1}{2} D_y^2 + \frac{3}{2}\lambda D_y - \frac{1}{2} D_x^2 D_y\right) f' \cdot f = 0, \tag{7c}$$

with λ as a constant.

Through BT (7), we can get the soliton solutions from the seed $f = 1$, i.e., $u = 0$. Setting $\lambda = 0$, we have

$$f' = e^{\xi} + e^{\zeta}, \tag{8}$$