Computers and Mathematics with Applications 73 (2017) 2454–2469

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Pricing European options with proportional transaction costs and stochastic volatility using a penalty approach and a finite volume scheme

Wen Li, Song Wang *

Department of Mathematics & Statistics, Curtin University, GPO Box U1987, Perth WA6845, Australia

ARTICLE INFO

Article history:
Received 3 October 2016
Accepted 26 March 2017
Available online 29 April 2017

Keywords:
Hamilton–Jacobi–Bellman equation
Financial option valuation
Finite volume method
Penalty method
Convergence

ABSTRACT

In this paper we propose a combination of a penalty method and a finite volume scheme for a four-dimensional time-dependent Hamilton–Jacobi–Bellman (HJB) equation arising from pricing European options with proportional transaction costs and stochastic volatility. The HJB equation is first approximated by a nonlinear partial differential equation containing penalty terms. A finite volume method along with an upwind technique is then developed for the spatial discretization of the nonlinear penalty equation. We show that the coefficient matrix of the discretized system is an M-matrix. An iterative method is proposed for solving the nonlinear algebraic system and a convergence theory is established for the iterative method. Numerical experiments are performed using a non-trivial model pricing problem and the numerical results demonstrate the usefulness of the proposed method.

1. Introduction

Valuation of options is one of the most important problems in financial engineering. For over four decades, practitioners and academic researchers in finance, economics and mathematics have engaged in the study of option pricing. Various option pricing approaches have been developed (see, for example, [1–9]). One of the methods is the utility based option pricing approach which has been widely used for valuing European and American options when the trading of the underlying stocks incurs proportional transaction costs [3,6,10–18]. Recently, Caflisch et al. [19] and Cosso [20] applied this approach to pricing European options and American options respectively under proportional transaction costs and stochastic volatility. More specifically, in [19] the authors assumed that the underlying stock price follows a geometric Brownian motion and the associated volatility evolves according to a stochastic process of the Ornstein–Uhlenbeck type. By following the utility maximization procedure proposed in [3], they derived a set of non-linear HJB equations governing European option prices. They also obtained an asymptotic expression for the European option price in the limit of small transaction costs and fast mean reversion volatility under the assumption of an exponential utility function. In [20], the authors considered American option pricing with proportional transaction costs and stochastic volatility. They assumed that the stochastic volatility follows the Cox–Ingersoll–Ross (CIR) process. They also showed that computing the price of an American option involves solving a singular stochastic optimal control problem and proved the existence and uniqueness of the viscosity solution to the associated HJB equation. Moreover, they solved the HJB equations using the Markov chain approximation when the utility function is exponential.

* Corresponding author.
E-mail addresses: wen.li@curtin.edu.au (W. Li), song.wang@uwa.edu.au (S. Wang).

http://dx.doi.org/10.1016/j.camwa.2017.03.024
0898-1221/

In this paper, we will develop a new, efficient and accurate numerical method for computing European option prices based on the pricing model in [19,20]. In both [19,20], the utility function is assumed to be an exponential function. It is well known that using the exponential utility function can reduce the number of state variables in the HJB equation by one under a proper transformation. Thus, the use of this special function can simplify the problem considerably. Although the transformation substantially reduces the computational cost, it may not be applicable to other types of utility functions. The aim of this paper is to develop a numerical method which can efficiently and accurately solve the HJB equation without any dimension reduction technique. Therefore, the numerical method developed in this work can be used for computing option prices with any types of utility function.

This paper is organized as follows. In Section 2, we give a brief account of the formulation of the European option valuation problem as a set of HJB equations using the utility maximization theory. In Section 3, we first use a known penalty approach to approximate the HJB equations by a nonlinear PDE with penalty terms to penalize the parts which violate the constraints. We then propose a finite volume scheme for the penalty equation. In Section 4, an iterative algorithm and its convergence will be provided and in Section 5, we present the numerical results to demonstrate the usefulness of the numerical method.

2. The European option pricing model

In this section, we will present a brief account of the European option pricing model when the volatility is stochastic and trading the underling stocks is subject to proportional transaction costs. A detailed mathematical deduction of the model can be found in [16,20].

2.1. Stochastic volatility model with transaction costs

Consider a market consisting of a risky stock and a risk-less bond. Assume that the price of the stock at time $u \in [0, T]$, denoted as S_u, evolves according to the following stochastic volatility model:

$$\frac{dS_u}{S_u} = \mu du + \sqrt{\nu(u)} dW_u^1, \tag{1}$$

where μ is constant drift rate and $\sqrt{\nu(u)}$ is the volatility function which satisfies the following Cox–Ingersoll–Ross (CIR) process:

$$d\nu(u) = \xi(\eta - \nu(u))du + \vartheta\sqrt{\nu(u)} dW_u^2, \tag{2}$$

where ξ is the speed of adjustment, η is the mean and ϑ is the volatility to volatility. In (2) ξ, η and ϑ are assumed to be constant satisfying $2\xi\eta > \vartheta^2$, and W_u^1 and W_u^2 are Wiener processes on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_u)_{0\le u\le T}, P)$ with correlation ρ.

We also assume that the price of the bond, $B(u)$, evolves according to the following ordinary differential equation

$$dB(u) = rB(u)du,$$

where $r \ge 0$ is a constant interest rate.

Suppose that the investors must pay transaction costs when buying or selling the stock and the transaction costs are proportional to the amount transferred from the stock to the bond. Let β_u denote the amount the investors hold in the bond and α_u the number of shares of the stock held by the investors at time $u \in [0, T]$, then the evolution equations for β_u and α_u are

$$d\beta_u = r\beta_u du - (1+\theta)S_u dL_u + (1-\theta)S_u dM_u, \tag{3}$$

$$d\alpha_u = dL_u - dM_u, \tag{4}$$

where $\theta \in [0, 1)$ represents the proportional transaction cost rate when buying and selling the stock, and L_u and M_u denote respectively the cumulative number of shares purchased and sold up to time u. Let $c(\alpha_u, S_u)$ denote the liquidated cash value of the stock and W_u the investor's wealth at time u. We have

$$c(\alpha_u, S_u) = S_u(\alpha_u - \theta|\alpha_u|)$$

$$W_u(\alpha_u, \beta_u, S_u) = \beta_u + S_u(\alpha_u - \theta|\alpha_u|).$$

2.2. European option pricing via utility maximization

We now describe the utility based option pricing approach. The idea of the utility based option pricing approach is to consider an optimal portfolio selection problem of an investor whose objective is to find an admissible trading strategy to maximize his/her expect utility of terminal wealth. Under this approach, the reservation purchase (respectively write) price of an option is the price at which the investor has the same maximum expected utility whether he/she buys (respectively writes) the option or not. To use this approach to value reservation purchase and write prices of European call options, we first need to define the following three different utility maximization problems.