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Hybrid Laplace transform and finite difference methods for pricing American options under complex models[☆]

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ABSTRACT

In this paper, we propose a hybrid Laplace transform and finite difference method to price (finite-maturity) American options, which is applicable to a wide variety of asset price models including the constant elasticity of variance (CEV), hyper-exponential jump–diffusion (HEJD), Markov regime switching models, and the finite moment log stable (FMLS) models. We first apply Laplace transforms to free boundary partial differential equations (PDEs) or fractional partial differential equations (FPDEs) governing the American option prices with respect to time, and obtain second order ordinary differential equations (ODEs) or fractional differential equations (FDEs) with free boundary, which is named as the early exercise boundary in the American option pricing. Then, we develop an iterative algorithm based on finite difference methods to solve the ODEs or FDEs together with the unknown free boundary values in the Laplace space. Both the early exercise boundary and the prices of American options are recovered through inverse Laplace transforms. Numerical examples demonstrate the accuracy and efficiency of the method in CEV, HEJD, Markov regime switching models and the FMLS models.

1. Introduction

Pricing American-style derivatives has attracted a lot of interest in the academia in the last thirty years (see [1] and [2]), and it is of practical importance to the industry, e.g. stock options may be exercised before contract expiration [3]. It is a classical yet challenging optimal stopping problem, especially when the underlying stock prices follow some jump–diffusion processes. The recent financial crisis highlights event risks (e.g. a market crash) as important fundamental elements for market modeling [4]. Thus, the inclusion of event risks or jumps in the valuation of American options are important to investors.

Jump–diffusion models capture the empirical facts of jumps and can be used to explain event risks and market crashes. There are two well-known jump–diffusion models in the literature: Merton's model (see [5]), and the double-exponential jump–diffusion (DEJD) model (see [6]), where the jump sizes follow normal and double exponential distributions, respectively. More general models include phase-type jump–diffusion model (see e.g., [7]), the hyper-exponential jump–diffusion (HEJD) model (see e.g., [8]), and the mixed-exponential jump–diffusion model (see e.g., [9]). The value of the American option under the HEJD model is governed by a partial integro-differential equation (PIDE) with a free boundary.

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The Markov regime switching models allow the model parameters (drift and volatility coefficients) to depend on a Markov chain that reflects the information of the market environments, and at the same time preserve the simplicity of the model. The first Markov regime switching model is introduced by [10]. It has been used for American option pricing. The paper [11] studies American option pricing by formulating the pricing problem into a system of partial differential equations (system PDEs) with free boundaries which are named as early exercise boundaries in the American option pricing, but there are no discussions on the solution of the system PDEs. The numerical methods for solving the system PDEs for pricing American options with regime switching models are studied in papers [12,13].

The finite moment log stable (FMLS) model assumes that the log value of the underlying asset follows a stochastic differential equation of the maximally skewed log stable process (see [14]). The option price under this kind of model is governed by fractional partial differential equations (see [15]). This kind of fractional partial differential equation is challenging to solve. Recently, the option pricing under the FMLS models has attracted some research interests (see [15]).

American option pricing can be regarded as a free boundary problem where the free boundary is referred to as the early exercise boundary or optimal exercise boundary in the mathematical finance literature (see [16]). Recently, Laplace transform methods have been developed to solve the free boundary problems arising in American option pricing under the geometric Brownian motion (GBM) model (see [17,18]), the constant elasticity of variance (CEV) model (see [19]) and the hyper-exponential jump diffusions (HEJD) model (see [20]). The essence of the Laplace transform methods can be regarded as a better alternative to the commonly used time-stepping scheme. The methods are described as follows: Applying Laplace transforms to the governing free boundary partial differential equations (PDEs) with respect to the time variable results in second-order ordinary differential equations (ODEs). Unlike the case for pricing European options, the ODEs involve a free boundary value[1] in the Laplace space which needs to be solved. If the solutions of the ODEs can be expressed in explicit form, then a nonlinear algebraic equation for the Laplace-space free boundary value can be derived. Using a simple solver for the nonlinear algebraic equation, one can get the free boundary value in the Laplace space. Consequently, the early exercise boundary and the value for the American option can be obtained through inverse Laplace transform. Thus, the Laplace transform methods crucially rely on the availability of explicit solutions of the resulting ODEs, and this limits the applicability of their method to just a handful of models, such as the CEV model (see e.g., [19]), and the HEJD model (see e.g., [20]). However, for some complex models governing the underlying asset prices, e.g. the Markov regime switching model, the resulting ODEs from the Laplace transform do not have explicit solutions. Thus, the standard Laplace transform methods cannot be applied to price American options in regime-switching models. Moreover, for the FMLS model, the American option price is governed by fractional partial differential equations (FPDEs) with free boundary. The Laplace transform of the free-boundary FPDEs leads to a fractional differential equation, which does not have explicit solution. Therefore, the standard Laplace transform methods cannot be used in this situation.

In this paper, we propose a hybrid Laplace transform and finite difference method (hybrid LT-FDM). We use finite difference methods to discretize the ODEs coupled with the approximation of the free boundary. The approximate free boundary value in the Laplace space is obtained from an iterative algorithm based on a discrete version of the smooth pasting condition, which is analogous to the smooth pasting condition in the continuous time as employed in [11,19,21], and [20]. The advantage of our method is that it does not require explicit knowledge of the solutions of the ODEs, thus rendering our method applicable to a wider class of underlying asset price models (e.g. regime switching models, finite activity jump models, FMLS models).

The remaining of the paper is organized as follows: in Section 2, the hybrid LT-FDM methods are presented under the CEV, HEJD, and Markov regime switching models. In Section 3, we compare the hybrid LT-FDM with the standard Laplace transform method under the CEV model (see [19]) and the HEJD model (see [20]), with the trinomial tree method under the Markov regime switching model (see [22]), and with the FDMs applied directly to the FPDEs under the FMLS model (see [15]). Numerical examples illustrate the accuracy and efficiency of the method. Section 4 concludes the paper.

2. Main results

2.1. The CEV model and LT-FDM

Assume that the underlying asset price is governed by the constant elasticity of variance (CEV) model (see e.g., [23])

$$dS_t = (r-q)S_t dt + \delta S_t^{\beta+1} dW_t, \tag{1}$$

where r is the risk-free interest rate, q is the dividend yield, and W_t is a standard Brownian motion. The term $\sigma(S) = \delta S^\beta$ represents the local volatility function and β can be interpreted as the elasticity of $\sigma(S)$, i.e., $\frac{d\sigma/\sigma}{dS/S} = \beta$. Here, δ is the scale parameter fixing the initial instantaneous volatility at time $t = 0$, and $\sigma_0 = \sigma(S_0) = \delta S_0^\beta$. If $\beta = 0$, then the SDE (1) becomes a geometric Brownian motion with the constant volatility rate $\sigma_0 = \delta$.

[1] Since we have taken the Laplace transform with respect to the time variable, the corresponding free boundary in the Laplace space does not depend on time and is an unknown constant.

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