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On the least squares generalized Hamiltonian solution of generalized coupled Sylvester-conjugate matrix equations ☆

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ABSTRACT

In this paper, we discuss the finite iterative algorithm to solve a class of generalized coupled Sylvester-conjugate matrix equations. We prove that if the system is consistent, an exact generalized Hamiltonian solution can be obtained within finite iterative steps in the absence of round-off errors for any initial matrices; if the system is inconsistent, the least squares generalized Hamiltonian solution can be obtained within finite iterative steps in the absence of round-off errors. Furthermore, we provide a method for choosing the initial matrices to obtain the minimum norm least squares generalized Hamiltonian solution of the system. Finally, numerical examples are presented to demonstrate the algorithm is efficient.

1. Introduction

Matrix equations are often encountered in control theory, stability analysis, perturbation analysis and some other fields of pure and applied mathematics. For example, in stability analysis of linear jump systems with Markovian transitions, the following matrix equations are typical coupled Lyapunov matrix equations

$$A_i^T + P_iA_i + Q_i + \sum_{j=1}^{n} \pi_{ij}P_j = 0, \quad i = 1, 2, \ldots, n, \tag{1.1}$$

where Q_i are positive definite matrices, π_{ij} are known transition probabilities and P_j are the unknown matrices [1,2]. When we calculate an additive decomposition of generalized transformation matrix equations [3], the following generalized Sylvester matrix equations

$$\begin{cases} AX - YB = E, \\ CX - YD = F \end{cases} \tag{1.2}$$

are often encountered, where X and Y are the matrices to be solved. Due to these applications, coupled matrix equations have been widely researched [4–15].

Sylvester-conjugate matrix equations is a kind of important matrix equations. Wu et al. [16] considered the following Yakubovich-conjugate matrix equations

$$X - A\overline{X}F = BY, \tag{1.3}$$

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where $X \in \mathbb{C}^{n\times p}$ and $Y \in \mathbb{C}^{r\times p}$ are the matrices to be determined. Karimi and Dehghen [17] considered the generalized coupled linear matrix equations of the form

$$\begin{cases} \sum_{j=1}^{l} A_{ij}X_jB_{ij} + \sum_{j=1}^{l} C_{ij}X_j^H D_{ij} = E_i, & i = 1, 2, \dots, s, \\ X_j \in C^{m_j\times n_j}, & j = 1, 2, \dots, l, \end{cases} \tag{1.4}$$

where $A_{ij}, C_{ij} \in \mathbb{C}^{p_i\times m_j}$, $B_{ij}, D_{ij} \in \mathbb{C}^{n_j\times q_i}$ and $E_i \in \mathbb{C}^{p_i\times q_i}$, $i = 1, 2, \dots, s, j = 1, 2, \dots, l$ are given matrices and $X_j \in \mathbb{C}^{m_j\times n_j}$, $j = 1, 2, \dots, l$ are unknown matrices to be determined. Note that if there exists some $1 \le j \le l$ such that both X_j and X_j^H are in one of Eqs. (1.4), then $m_j = n_j$. They proved the minimum-norm solution can be obtained when the matrix equations are consistent and the optimal approximation solution to a given group of matrices can be derived. Recently, Huang and Ma [6] used the conjugate gradient method for obtaining the minimum norm solution of the following generalized coupled Sylvester-conjugate matrix equations

$$\begin{cases} A_1X + B_1Y = D_1\overline{X}E_1 + F_1, \\ A_2X + B_2Y = D_2\overline{X}E_2 + F_2, \end{cases} \tag{1.5}$$

where $A_1, B_1, D_1, A_2, B_2, D_2 \in \mathbb{C}^{p\times m}$, $E_1, E_2 \in \mathbb{C}^{n\times n}$, $F_1, F_2 \in \mathbb{C}^{p\times n}$ are given matrices and $X, Y \in \mathbb{C}^{m\times n}$ are unknown matrices that need to be solved.

Although there are all kinds of iterative methods (see [3,18–32]) for solving the matrix equations, few researchers considered the least squares solution of the matrix equations (see [33–37]) when the matrix equations are inconsistent. Particularly, the least squares generalized Hamiltonian solutions of the matrix equations are still open. For this reason, we consider the generalized coupled Sylvester-conjugate matrix equations of the form

$$\sum_{j=1}^{l} A_{ij}X_jB_{ij} + \sum_{j=1}^{l} C_{ij}\overline{X}_jD_{ij} = E_i, \quad i = 1, 2, \dots, s, \tag{1.6}$$

where $A_{ij}, C_{ij} \in \mathbb{C}^{m\times n}$, $B_{ij}, D_{ij} \in \mathbb{C}^{n\times r}$ and $E_i \in \mathbb{C}^{m\times r}$, $i = 1, 2, \dots, s, j = 1, 2, \dots, l$, are given matrices and $X_j \in \mathbb{C}^{n\times n}$, $j = 1, 2, \dots, l$, are unknown matrices to be determined. Similar to the previous works, we propose a finite iterative algorithm to solve the system (1.6). We consider two cases. Moreover, we prove that if the system (1.6) is consistent, an exact generalized Hamiltonian solution $(X_1^*, X_2^*, \dots, X_l^*)$ can be obtained within finite iterative steps in the absence of round-off errors for any initial matrices. If the system (1.6) is inconsistent, the least squares generalized Hamiltonian solution of the system (1.6) can be obtained within finite iterative steps in the absence of round-off errors; moreover, the minimum norm least squares generalized Hamiltonian solution can be derived by finding the special type of the initial matrices.

For convenience, we use the following notations throughout this paper. Let $\mathbb{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ be the sets of all real and complex $m \times n$ matrices, respectively. We abbreviate $\mathbb{C}^{n\times 1}$ as $\mathbb{C}^n$. For $A \in \mathbb{C}^{m\times n}$, we write $\overline{A}, A^T, A^H, \|A\|_F, tr(A), A^{-1}$ and $\mathcal{R}(A)$ to denote conjugation, transpose, conjugate transpose, Frobenius norm, the trace, the inverse and the column spaces of matrix A, respectively. For any matrices $A = (a_{ij})$, $B = (b_{ij})$, matrix $A \bigotimes B$ denotes the Kronecker product defined as $A \bigotimes B = (a_{ij}B)$. For the matrix $X = (x_1, x_2, \dots, x_n) \in \mathbb{R}^{n\times n}$, $vec(X)$ denotes the vec operator defined as $vec(X) = (x_1^T, x_2^T, \dots, x_n^T)^T \in R^{mn}$. Let $OASR^{n\times n}$ be the sets of all $n \times n$ real orthogonal antisymmetric matrices, i.e.,

$$OASR^{n\times n} = \{J | J^TJ = JJ^T = I_n, J = -J^T, \ J \in \mathbb{R}^{n\times n}\}.$$

It is clear that $J^2 = -I_n$, $\forall J \in OASR^{n\times n}$. Consequently, n must be an even integer. Let $HC^{n\times n}$ be the sets of all $n \times n$ generalized Hamiltonian matrices (see [38]), i.e.,

$$HC^{n\times n} = \{A | JAJ = A^H, \ A \in \mathbb{C}^{n\times n}\}, \tag{1.7}$$

where $J \in OASR^{n\times n}$.

The rest of this paper is organized as follows. In Section 2, we construct a finite iterative algorithm for solving the system (1.6) and prove that if the system is consistent, an exact generalized Hamiltonian solution $(X_1^*, X_2^*, \dots, X_l^*)$ can be obtained within finite iterative steps in the absence of round-off errors for any initial matrices. Moreover, we derive that if the system is inconsistent, the least squares generalized Hamiltonian solution can be obtained within finite iterative steps in the absence of round-off errors. In Section 3, we provide a method for choosing the initial matrices to obtain the minimum norm least squares generalized Hamiltonian solution of the system (1.6). In Section 4, we present some numerical experiments. Finally, we give our conclusions in Section 5.

2. Iterative algorithm for solving Eq. (1.6)

First, we give the definition of the inner product from [5,39]. In the space $\mathbb{C}^{m\times n}$ over the field $\mathbb{R}$, the inner product can be defined as

$$\langle A, B\rangle = Re[tr(A^HB)]. \tag{2.1}$$