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Characteristic block-centered finite difference method for simulating incompressible wormhole propagation

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ABSTRACT

In this paper, the characteristic block-centered finite difference method is introduced and analyzed to solve the incompressible wormhole propagation. Error estimates for the pressure, velocity, porosity, concentration and its flux in different discrete norms are established rigorously and carefully on non-uniform grids. Finally, some numerical experiments are presented to show that the convergence rates are in agreement with the theoretical analysis.

1. Introduction

Wormhole formation during reactive dissolution of carbonates is a subject that has been widely studied in the recent years. And it plays a very important role in the product enhancement of oil and gas reservoir. It is matrix acidization that plays a significant role in enhancing oil production rate when muds and fines deposit at the perforated well pore pipe [1–6]. The primary objective of this process is to increase the production rate by increasing permeability of the damaged zone near the well bore. The injected acid dissolves the material near the well bore and creates flow channels that establish a good connectivity between the reservoir and the well. Such channels with high porosities are called wormholes. The combination of the finite element method and the finite difference method has been studied to simulate the wormhole generation and propagation [7]. And in [1], mixed finite element-based fully conservative methods to simulate the wormhole propagation has been developed by Kou and his coauthors.

Block-centered finite differences, sometimes called cell-centered finite differences, can be thought as the lowest order Raviart–Thomas mixed element method [8], with proper quadrature formulation. In [9], Wheeler presented the mixed finite elements for elliptic problems with tensor coefficients as cell-centered finite differences. And in 2012, a block-centered finite difference method for the Darcy–Forchheimer model was considered [10]. In [11–14] block-centered finite difference methods were developed. Recently, a parallel CGS block-centered finite difference method for a nonlinear time-fractional parabolic equation has been studied [15]. Besides, Li and Sun [16] studied a new block-centered finite element method, sometimes called marker-and-cell (MAC) method for Stokes problem, which achieved superconvergence phenomenon. We consider block-centered finite differences in this paper since these schemes are of practical use in implementation and can achieve superconvergence.

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As far as we know, there is no characteristic block-centered finite difference method for incompressible wormhole propagation. Clearly, the motivation for including the method of characteristics is to follow the transport more accurately than the standard finite difference method does. Before analyzing the convergence of our characteristic block-centered finite difference method, we develop estimates of the expanded mixed finite element method with quadrature applied to linear parabolic equations. Using these estimates, we obtain the superconvergence of the pressure, velocity, porosity, concentration and its flux in different discrete norms. The error estimates are deduced rigorously and carefully in this paper, and we carry out some numerical experiments to show that the convergence rates are in agreement with the theoretical analysis.

The paper is organized as follows. In Section 2 we give the problem and some preliminaries. In Section 3 we present the characteristic block-centered finite difference algorithm. In Section 4 we demonstrate error analysis for the discrete scheme. In Section 5 some numerical experiments using the characteristic block-centered finite difference scheme are carried out, which show that the convergence rates are in agreement with the theoretical analysis.

Through out the paper we use C, with or without subscript, to denote a positive constant, which could have different values at different appearances.

2. The problem and some preliminaries

In this paper, we consider the usual differential system used to describe incompressible wormhole propagation given by [1,2]

$$\begin{cases} \dfrac{\partial \phi}{\partial t} + \nabla \cdot \mathbf{u} = f, \quad \mathbf{x} \in \Omega,\ t \in J, \quad \text{(a)} \\ \mathbf{u} = -\dfrac{K(\phi)}{\mu} \nabla p, \quad \mathbf{x} \in \Omega,\ t \in J, \quad \text{(b)} \\ \dfrac{\partial (\phi c_f)}{\partial t} + \nabla \cdot (\mathbf{u} c_f) - \nabla \cdot (\phi \mathbf{D} \nabla c_f) = k_c a_v (c_s - c_f) + f_P c_f + f_I c_I, \quad \mathbf{x} \in \Omega,\ t \in J, \quad \text{(c)} \\ \dfrac{\partial \phi}{\partial t} = \dfrac{\alpha k_c a_v (c_f - c_s)}{\rho_s}, \quad \mathbf{x} \in \Omega,\ t \in J, \quad \text{(d)} \end{cases} \tag{1}$$

where Ω is a rectangular domain in $\mathbb{R}^2$. $J = (0, \hat{T}]$, and $\hat{T}$ denotes the final time. p is the pressure, μ is the fluid viscosity, $\mathbf{u}$ is the Darcy velocity of the fluid, $f = f_I + f_P$, f_P and f_I are production and injection rates respectively. The first term in the continuity equation (1)(a) accounts for the effect of local volume change during dissolution on the flow field. c_f is the cup-mixing concentration of the acid in the fluid phase. c_I is the injected concentration. For simplicity we assume that diffusion coefficient $\mathbf{D}(\mathbf{x}) = d_{mol}\mathbf{I} = \text{diag}(D_{ll})$, $l = 1, 2$ is diagonal matrix in the following. k_c is the local mass-transfer coefficient, a_v is the interfacial area available for reaction per unit volume of the medium. Eq. (1)(d) describes the evolution of porosity in the domain due to reaction. The variable c_s is the concentration of the acid at the fluid–solid interface, and the relationship between c_f and c_s can be described as follows:

$$c_s = \frac{c_f}{1 + k_s/k_c}, \tag{2}$$

where k_s is the surface reaction rate constant. The three terms on the left hand side of Eq. (1)(c) represent the accumulation, convection and dispersion of the acid respectively. The first term on the right hand side of Eq. (1)(c) represents the transfer of acid species from the fluid phase to the fluid–solid interface [2]. ϕ and K are the porosity and permeability of the rock respectively, the relationship between the permeability and the porosity is established by the Carman–Kozeny correlation [17]

$$\frac{K}{K_0} = \frac{\phi}{\phi_0} \left(\frac{\phi(1 - \phi_0)}{\phi_0(1 - \phi)} \right)^2, \tag{3}$$

where ϕ_0 and K_0 are the initial porosity and permeability of the rock respectively. In Eq. (1)(d), α is the dissolving power of the acid and ρ_s is the density of the solid phase. Using porosity and permeability, a_v is shown as

$$\frac{a_v}{a_0} = \frac{\phi}{\phi_0} \sqrt{\frac{K_0 \phi}{K \phi_0}}, \tag{4}$$

where a_0 is the initial interfacial area.

The boundary and initial conditions are as follows.

$$\begin{cases} \mathbf{u} \cdot \mathbf{n} = 0, \qquad \phi \mathbf{D} \nabla c_f \cdot \mathbf{n} = 0, \quad \mathbf{x} \in \partial\Omega,\ t \in J, \\ c_f(\mathbf{x}, 0) = c_0(\mathbf{x}), \quad \mathbf{x} \in \Omega, \\ \phi(\mathbf{x}, 0) = \phi_0(\mathbf{x}), \quad \mathbf{x} \in \Omega, \end{cases} \tag{5}$$