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Existence of random attractors for weakly dissipative plate equations with memory and additive white noise

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ABSTRACT

In this paper, we prove the existence of random attractors for the continuous random dynamical systems generated by stochastic weakly dissipative plate equations with linear memory and additive white noise by defining the energy functionals and using the compactness translation theorem.

1. Introduction

In this paper, we are devoted to consider the existence of random attractors for the following plate equations with linear memory and additive white noise:

$$\begin{cases} u_{tt} + \Delta^2 u + \int_0^\infty \mu(s)\Delta^2(u(t) - u(t-s))ds + g(u) = f(x) + \sum_{j=1}^m h_j \dot{W}_j, & x \in U,\ t \geq 0, \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), & x \in U,\ t \leq 0, \\ u|_{\partial U} = \left.\dfrac{\partial u}{\partial \mathbf{n}}\right|_{\partial U} = 0, & t \geq 0, \end{cases} \tag{1.1}$$

where $u = u(x,t)$ is a real valued function on $U \times [0,+\infty)$, $f(x) \in H_0^2(U)$ is a given external force. $h_j(x) \in H_0^2(U) \cap H^4(U)$ $(j = 1,2,3,\ldots,m)$, $\{W_j\}_{j=1}^m$ are independent two-sided real-valued Wiener processes on a probability space $(\Omega, \mathscr{F}, \mathbb{P})$, where

$$\Omega = \{\omega = (\omega_1, \omega_2, \ldots, \omega_m) \in C(\mathbb{R}, \mathbb{R}^m) : \omega(0) = 0\}$$

is endowed with compact open topology, $\mathbb{P}$ is the corresponding Wiener measure, and $\mathscr{F}$ is the $\mathbb{P}$-completion of Borel σ-algebra on Ω. We identify $\omega(t)$ with $(W_1(t), W_2(t), \ldots, W_m(t))$, i.e.,

$$\omega(t) = (W_1(t), W_2(t), \ldots, W_m(t)), \quad t \in \mathbb{R}.$$

Then, define the time shift by

$$\theta_t\omega(\cdot) = \omega(\cdot + t) - \omega(t), \quad t \in \mathbb{R},\ \omega \in \Omega.$$

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The memory function $\mu(s)$ and the nonlinear term $g \in C^2(\mathbb{R}, \mathbb{R})$ satisfy the following conditions:
(h_1): $\mu(s) \in C^1(\mathbb{R}^+) \cap L^1(\mathbb{R}^+),\ \mu'(s) \leq 0 \leq \mu(s),\ \forall s \in \mathbb{R}^+$;
(h_2): $\int_0^\infty \mu(s)ds = \mu_0$;
(h_3): $\mu'(s) + \delta\mu(s) \leq 0,\ \forall s \in \mathbb{R}^+$ and some $\delta > 0$;
(h_4): There exists $s_0 > 0$, such that $\mu'(s) \in L^2((0, s_0)),\ \mu'(s) + M\mu(s) \geq 0,\ \forall s \geq s_0$ and some $M > 0$;
(g_1): $\liminf_{|s|\to\infty} \frac{G(s)}{s^2} \geq 0$;
(g_2): $\liminf_{|s|\to\infty} \frac{(s,g(s))-C_0G(s)}{s^2} \geq 0,\ \forall s \in \mathbb{R}^+$;
(g_3): $|g'(s)| \leq l,\ g(0) = 0,\ \forall s \in \mathbb{R}$,
where $G(s) = \int_0^s g(\tau)d\tau$, and the following inequalities are direct consequences of (g_1)–(g_2),

$$G(u) + \frac{1}{4}\|u\|_2^2 \geq -C_1, \quad \forall u \in H_0^2(U), \tag{1.2}$$

$$\int_U ug(u)dx - C_0G(u) + \frac{1}{4}\|u\|_2^2 \geq -C_2, \quad \forall u \in H_0^2(U), \tag{1.3}$$

for some $C_1,\ C_2 > 0$. Following Dafermos [1], we introduce a Hilbert "history" space $\mathfrak{R}_{\mu,2} = L^2_\mu(\mathbb{R}^+, H_0^2(U))$, with the inner product

$$(\eta_L, \eta_N)_{\mu,2} = \int_0^\infty (\Delta\eta_L(s), \Delta\eta_N(s))\, ds, \quad \forall \eta_L, \eta_N \in \mathfrak{R}_{\mu,2},$$

and new variables

$$\eta(x, t, s) = u(t, x) - u(t - s, x).$$

Set $E = H_0^2(U) \times L^2(U) \times \mathfrak{R}_{\mu,2},\ Z = (u, u_t, \eta)^T$, then the system (1.1) is equivalent to the following initial value problem in the Hilbert space E:

$$\begin{cases} Z_t = L(Z) + N(Z, t, W(t)), & x \in U,\ t \geq 0, \\ Z_0 = (u_0(x), u_1(x), \eta_0(x, s)), & (x, s) \in U \times \mathbb{R}^+, \end{cases} \tag{1.4}$$

where

$$\begin{cases} u(t, x) = \eta(t, x, s) = \eta(t, x, 0) = 0, & x \in \partial U,\ t \geq 0,\ s \in \mathbb{R}^+, \\ u(0, x) = u_0(x),\ u_t(0, x) = u_1(x), & x \in U, \\ \eta(0, x, s) = \eta_0(x, s) = u_0(0, x) - u_0(-s, x), & (x, s) \in U \times \mathbb{R}^+, \end{cases} \tag{1.5}$$

$$L(Z) = \begin{pmatrix} u_t \\ -\Delta^2 u - \int_0^\infty \mu(s)\Delta^2\eta(s)ds \\ u_t - \eta_s \end{pmatrix}, \tag{1.6}$$

$$N(Z, t, W(t)) = \begin{pmatrix} 0 \\ -g(u) + f(x) + \sum_{j=1}^m h_j\dot{W}_j \\ 0 \end{pmatrix}, \tag{1.7}$$

$$D(L) = \left\{ Z \in E \,\middle|\, \begin{matrix} u + \int_0^\infty \mu(s)\eta(s)ds \in H^4(U) \cap H_0^2(U), \\ u_t \in H_0^2(U), \quad \eta(s) \in H^1_\mu(\mathbb{R}^+, H_0^2(U)), \eta(0) = 0 \end{matrix} \right\}, \tag{1.8}$$

here $H^1_\mu(\mathbb{R}^+, H_0^2(U)) = \{\eta : \eta(s), \partial_s\eta(s) \in L^2_\mu(\mathbb{R}^+, H_0^2(U))\}$.

Problem (1.1) models transversal vibrations of thin extensible elastic plate in a history space, which is established based on the framework of elastic vibration by Woinowsky-Krieger [2] and Berger[3]. It can also be regarded as an elastoplastic flow equation with some kind of memory effect[1].

If we consider the linear damping in (1.1), the following stochastic plate equation is achieved

$$u_{tt} + \alpha u_t + \Delta^2 u + \int_0^\infty \mu(s)\Delta^2(u(t) - u(t - s))ds + g(u) = f(x) + \sum_{j=1}^m h_j\dot{W}_j. \tag{1.9}$$

When $h_j = 0$ $(1 \leq j \leq m)$ and $\mu = 0$, Eq. (1.9) reduces to a normal determined autonomous damped plate equation. There were a lot of publications concerning the existence of their random attractors, uniform attractors, pullback attractors and exponential attractors, see for instance [4–9]. When $h_j \neq 0$ $(1 \leq j \leq m)$ and $\mu = 0$, Eq. (1.9) transforms into a

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