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An analytic study of opinion dynamics in multi-agent systems

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ABSTRACT

This paper presents an analytic approach that can be used to study opinion dynamics in multi-agent systems. The results of such an analytic approach can be used as a *descriptive* tool capable of predicting the long-term properties of a multi-agent system, and they can also be considered a *prescriptive* tool that supports the design of multi-agent systems with desired asymptotic characteristics. The agents that form the multi-agent system are divided into disjoint classes characterized by different values of fixed parameters to account for the specific behaviors of single agents. Each class is characterized by the number of agents in it, by the initial distribution of the opinion, and by the characteristic propensity of single agents to change their respective opinions when interacting with other agents. The proposed approach is based on the possibility of interpreting the dynamics of the opinion in terms of the kinetic theory of gas mixtures, which allows expressing the dynamics of the average opinion of each class in terms of a suitable differential problem that can be used to derive interesting asymptotic properties. Analytic solutions of the obtained differential problem are derived and it is shown that, under suitable hypotheses, the average opinions of all classes of agents converge to the same value. The results presented in this paper differ from those commonly derived in standard kinetic theory of gas mixtures because the microscopic equations which describe the effects of interactions among agents are explicitly meant to model opinion dynamics, and they are different from those normally used to describe collisions among molecules in a gas. All presented analytic results are confirmed by simulations presented at the end of the paper.

1. Introduction

Opinion dynamics and consensus formation deal with the identification of interaction rules which lead to collective cognitive convergence or to proper opinion distribution in a multi-agent system [1]. They are important topics of the research on multi-agent systems and distributed computing, and they have applications in various fields, such as control theory, robotics, biology, and sociology (e.g., [2,3]). Consensus formation, in particular, is typically relevant to distributed and cooperative multi-agent systems and to social networks (e.g., [4,5]). In order to describe the effects of interactions among agents, various approaches have been proposed in the literature, among which we can recall those based on thermodynamics (e.g., [6]), on Bayesian networks [7], and on gossip-based algorithms [7]. The use of cellular automata to model consensus formation has also been investigated; in this case, opinion is modeled as a discrete variable and consensus is reached through proper transition rules [8]. Another important framework which is useful to study opinion and consensus formation is related to graph theory and properties of graph Laplacians [9,10]. Finally, it is worth noting that opinion dynamics is studied fruitfully in [11] according to a flocking model.

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In this paper we consider a population of agents and we assume that each one of them is associated with a parameter representing its opinion. We aim at analytically studying the evolution of such a parameter under the assumption that each agent can change its opinion only after interacting with another agent. We also assume that each agent can interact with any other agent in the multi-agent system, and that each interaction is binary, namely that it involves only two agents [12]. Notably, the assumption of binary interactions is not restrictive because single interactions involving many agents can be often split into independent binary interactions. We also assume that the considered population of agents is divided into disjoint classes that are associated with specific characteristics. The most important of such characteristics is that agents belonging to different classes have different propensity to change their opinions when interacting with other agents. Since we allow singleton classes, the proposed model can be used to describe a population where all agents have a different propensity to change opinion. Finally, we model time as a sequence of discrete steps and each step corresponds to exactly one interaction among two agents. Steps are not assumed to be of equal duration, and the model associates a time step to each interaction event, whenever it may happen [13]. The details of the rules that model the post-interaction opinions as functions of pre-interaction opinions are described in next section, and they form the basis of the studied analytic model.

The analytic framework that we adopt to study the dynamics of the opinion in multi-agent systems is derived from a model based on the kinetic theory of gases [14], which normally studies microscopic interactions among the molecules in a gas. In particular, the model adopted in this paper is based on the kinetic theory of gas mixtures, but the presented results differ from those of standard kinetic theory of gas mixtures because we propose microscopic equations which are meant explicitly to model the dynamics of the opinion, and which are different from those that describe collisions among molecules in a gas. In recent years, models based on kinetic theory have been applied to very diverse research domains to describe populations of agents which coexist and interact in a given environment. The molecules of a gas can be considered as a population of agents, and the physical rules which describe the effects of the collisions among molecules can be reinterpreted to describe interactions among agents. In the early 1990s, the term *sociophysics* was introduced to describe an interdisciplinary research field aimed at applying methods originally developed by physicists in various contexts [15]. For example, in [16], the similarity between the distribution of wealth in a simple economy and the density of molecules in a gas is studied. In [17,18], kinetic models of opinion dynamics are studied, and they are applied in the field of multi-agent systems in [19,20]. The major difference between the model presented in this paper and the results of such – and many other – works is that previous literature does not account for different classes of agents, and therefore it does not account for agents with specific characteristics.

The major advantage that we expect from the adoption of an analytic framework like kinetic theory is that analytic derivations of the properties of multi-agent systems can be equally used as *descriptive* and as *prescriptive* tools. As a descriptive tool, the analytic approach that we develop can be used as an alternative to simulation. The results obtained via simulations are typically influenced by the actual values of the parameters used in simulation, especially for large multi-agent systems. On the contrary, provided that underlying assumptions are valid, analytic results are valid. As a prescriptive tool, the proposed analytic approach can support the design of multi-agent systems with desired properties because the analytic formulations can be used to identify the actual values of the specific parameters to have the multi-agent system behave as intended.

This paper is organized as follows. Section 2 presents an overview of classic kinetic theory, focusing on kinetic models of gas mixtures and fixing the needed notation. Section 3 describes the proposed model of opinion dynamics. Section 4 reports analytic and simulation results which confirm the behaviors predicted by the proposed analytic model. Finally, Section 5 concludes the paper.

2. Classic kinetic theory and notation

Classic kinetic theory describes the behavior of gases by considering the effects of the interactions among molecules (or atoms for noble gases) and it explains macroscopic properties of gases starting from microscopic interactions. The foundational approach of kinetic theory is to describe the microscopic collisions among the molecules of a gas from a probabilistic point of view, describing collective behaviors by means of a proper balance equation. Such an equation can have many forms, depending on considered details of collisions. Typically, it is assumed that the behavior of a gas is described according to the Boltzmann equation, which is an integro-differential equation whose unknown is the function $f(\underline{x}, \underline{v}, t)$, which is a non-negative function that represents the density of the molecules at position $\underline{x} \in \mathbb{R}^3$ with velocity $\underline{v} \in \mathbb{R}^3$ at time $t \geq 0$. The spatially homogeneous formulation of the Boltzmann equation is

$$\frac{\partial f}{\partial t}(\underline{v}, t) = \mathcal{Q}(f, f)(\underline{v}, t) \tag{1}$$

where the right-hand side represents the effects of binary interactions among molecules and, for this reason, $\mathcal{Q}(f, f)$ is known as *collisional operator*.

Observe that the formulation of the Boltzmann Eq. (1) is relative to a gas where all molecules are equal. Instead, a gas is typically made of different kinds of molecules. For this reason, it is of interest to study the behavior of *gas mixtures*, namely of gases with different kinds of molecules, typically called *species*. Denoting as m the number of species in the considered gas, m distribution functions $\{f_s\}_{s=1}^m$ need to be considered (one for each species). When considering a gas mixture, the temporal

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