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Stability and bifurcation of a ratio-dependent prey–predator system with cross-diffusion☆

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ABSTRACT

This paper is purported to investigate a ratio-dependent prey–predator system with cross-diffusion in a bounded domain under no flux boundary condition. The asymptotical stabilities of nonnegative constant solutions are investigated to this system. Moreover, without estimating the lower bounds of positive solutions, the existence, multiplicity of positive steady states are considered by using fixed points index theory, bifurcation theory, energy estimates and the differential method of implicit function and inverse function.

1. Introduction

When the predators invade the high level of concentration of preys to predate, the preys run away or switch to group defense. These intricate movements give rise to cross-diffusion which describes the flux of species caused from the mutual interferences. The movement of the species is in the direction of lower concentration of another species when the cross-diffusion coefficient is positive and the tendency of the population is in the direction of higher concentration of another species when the cross-diffusion coefficient is negative. These general partial differential predator–prey systems with cross-diffusion were put forward by the papers [1,2].

In [3], the authors prove that there exists globally a unique classical solution of the following cross-diffusion predator–prey system.

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta(d_1 u + d_2 u^2) = u\Big(a - u - \dfrac{v}{u+v}\Big) & \text{in } (0,T)\times\Omega, \\ \dfrac{\partial v}{\partial t} - \Delta\Big(d_3 v + d_4 v^2 + \dfrac{d_5 v}{1+u}\Big) = v\Big(-b + \dfrac{eu}{u+v}\Big) & \text{in } (0,T)\times\Omega, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } (0,T)\times\partial\Omega, \\ (u(0,x), v(0,x)) = (u_0(x), v_0(x)) \geq (0,0) & \text{in } \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in R^N ($N \geq 1$ is an integer) with a smooth boundary $\partial\Omega$; u and v represent the densities of the prey and predator respectively; the positive constants a, b, e represent the prey intrinsic growth rate, the death rate

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of the predator and the conversion rate respectively. In diffusion terms, d_1 and d_3 represent the natural dispersive force of movement of individual, while d_5 describes the mutual interferences between predators and preys. d_5 is also called cross-diffusion pressures. d_2 and d_4 represent self-pressure. The model (1.1) means that, in addition to the dispersive force, the diffusion also depends on population pressure from other species.

The predators of the system (1.1) diffuse with the flux

$$
\begin{aligned}
J &= -\nabla\left(d_3 v + d_4 v^2 + \frac{d_5 v}{1+u}\right) \\
&= -(d_3 + 2d_4 v + d_5/(1+u))\nabla v + \frac{d_5 v}{(1+u)^2}\nabla u.
\end{aligned}
$$

The part $-(d_3 + 2d_4 v + d_5/(1+u))\nabla v$ of the flux is directed toward the decreasing population densities of the predators, which indicates that preys run away from the predators to avoid being caught. The part $\frac{d_5 v}{(1+u)^2}\nabla u$ of the flux is directed toward the increasing population densities of the prey, which indicates that predators move towards the preys to catch [4].

As the predators move into areas with high food abundance to increase the efficiency of foraging, and the preys run away or switch to defend. These movements are quite complicated and called cross-diffusion which demonstrates rich dynamics. Coexistence and stability of the predator–prey system are interesting problems. Usually, the species can coexist in nonconstant positive steady states, while under certain conditions the constant positive steady states are stable. Many authors have established the coexistence and stability in various population dynamics models [5–13], but most of them are without cross-diffusion. To my knowledge, this is the first to use differential method of implicit function and inverse function investigating cross-diffusion system. In paper [14], the authors consider the positive steady state solutions of a Leslie–Gower predator–prey model which includes cross-diffusion only in the equation of predator w. In paper [15], the authors also consider the positive steady state solutions of the model with cross-diffusion only in the equation of the prey w. In these two papers, the authors investigated the models by changing them into the systems only including random diffusion, so they did not employ differential method of implicit function and inverse function.

Although, in paper [3], the authors investigated the existence of classical solution in $C^{2+\alpha,1+\alpha/2} \times C^{2+\alpha,1+\alpha/2}(0 < \alpha < 1)$ to the system (1.1), there were no studies in its steady states. In the following sections we mainly investigate steady states of the system (1.1) including stability of constant steady states and existence of nonconstant steady states, which is continuous work of paper [3].

The corresponding elliptic system of (1.1) is the following

$$
\begin{cases}
-\Delta(d_1 u + d_2 u^2) = u\left(a - u - \dfrac{v}{u+v}\right) & \text{in } \Omega, \\
-\Delta\left(d_3 v + d_4 v^2 + \dfrac{d_5 v}{1+u}\right) = v\left(-b + \dfrac{eu}{u+v}\right) & \text{in } \Omega, \\
\dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0 & \text{on } \partial\Omega.
\end{cases}
\tag{1.2}
$$

This paper is organized as follows. In Section 2, the stabilities of steady states of (1.1) are investigated. In Section 3, by using fixed points index theory the existence of non-constant steady states of (1.2) is analyzed and nonexistence of non-constant steady states is considered. In Section 4, the bifurcation and multiplicity of positive solutions are considered. In the final section, the conclusion and discussion are derived.

2. Stability analysis

In this section we shall study the stability of positive constant steady states of (1.1).

The system (1.1) admits the following three non-negative constant solutions:

(i) the extinction equilibrium state (0, 0);
(ii) the semi-trivial equilibrium state $(a, 0)$;
(iii) the unique positive equilibrium state $\mathbf{w}_* =: (u_*, v_*)$, where

$$
u_* = a - 1 + \frac{b}{e}, \qquad v_* = \frac{(e-b)(ea-e+b)}{eb}.
$$

If $e > b$ and $b < e < ea+b$, then the positive equilibrium state exists for system (1.1). Therefore, for existence of positive constant solution, we assume in whole paper that

(**H**) $e > b$ and $b < e < ea + b$.

It is easy to check that

$$
\begin{aligned}
&\Delta(d_1 u + d_2 u^2) = d_1 \Delta u + 2d_2 |\nabla u|^2 + 2d_2 u \Delta u, \\
&\Delta\left(d_3 v + d_4 v^2 + \frac{d_5 v}{1+u}\right) = d_3 \Delta v + 2d_4 |\nabla v|^2 + 2d_4 v \Delta v + \frac{d_5}{1+u}\Delta v \\
&\qquad - \frac{2d_5}{(1+u)^2}\nabla u \cdot \nabla v + \frac{2d_5 v}{(1+u)^3}|\nabla u|^2 - \frac{d_5 v}{(1+u)^2}\Delta u.
\end{aligned}
$$

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