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Boundary data identification for an electromagnetic problem by means of the potential field method ☆

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ABSTRACT

This paper is devoted to the study of a boundary data identification for an electromagnetic problem by means of the potential field method (the $\boldsymbol{A}$-ϕ method). One part of the boundary is over-determined. The other part of the boundary is unreachable and has to be determined as a part of the problem. We design a constructive algorithm by the $\boldsymbol{A}$-ϕ formulation to solve this problem. The numerical scheme is based on the steepest descent method (SDM) for the minimization of a regularized cost functional, having its derivative determined via an adjoint method. We analyze the properties of the cost functional and prove the convergence of the minimization process. The method is supported by several numerical experiments.

1. Introduction

In this paper we study an identification electromagnetic problem with an unknown boundary data and determine its solution inside a domain from the measurements on a part of the boundary. The lack of information on the unreachable part of the boundary is compensated by over-determined data on the reachable part. A practical application of the problem is the so-called passive shielding, see [1]. Induction heating equipment is employed for a thermal treatment of a metallic specimen. This thermal treatment is achieved by huge eddy currents, induced in the conductive piece. This gives rise to magnetic field levels that may be significantly higher than the allowed reference levels. Passive shielding is one of the techniques used to minimize the magnetic field inside a given area, and one side of the shield may not be accessible for measurements.

Let $\Omega \subset \mathbb{R}^3$ be a bounded convex polyhedron. The boundary Γ is split in N (open) faces $\Gamma_j, j = 1, \dots, N, \Gamma = \cup_j \overline{\Gamma}_j$. Possible current sources in Ω are expressed by the source term $\boldsymbol{J}_s$. Electromagnetic fields in this domain are described by Maxwell's equations

$$\begin{cases} \nabla \times \boldsymbol{H} = \sigma \boldsymbol{E} + \boldsymbol{J}_s + \partial_t(\varepsilon \boldsymbol{E}) & \text{in } \Omega \times (0, T), \\ \nabla \times \boldsymbol{E} = -\partial_t(\mu \boldsymbol{H}) & \text{in } \Omega \times (0, T), \end{cases} \tag{1.1}$$

accompanied by standard boundary conditions like

$$\boldsymbol{E} \times \boldsymbol{n} = \widetilde{\boldsymbol{C}}_E \quad \text{or} \quad \boldsymbol{H} \times \boldsymbol{n} = \widetilde{\boldsymbol{D}}_H,$$

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where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field, ε, μ and σ denote the electric permittivity, the magnetic permeability and the electric conductivity of the medium. Let $\boldsymbol{n}$ denote the unit outward normal to Ω.

The Maxwell system is usually transformed to the $\boldsymbol{E}$ or $\boldsymbol{H}$ equation and may be solved approximately by edge finite element methods. Besides, it can also be changed into potential formulations by means of decomposition of the field $\boldsymbol{E}$ or $\boldsymbol{H}$ (called the $\boldsymbol{A}$-ϕ or $\boldsymbol{T}$-ψ method) and thus nodal finite elements are used to solve it numerically, cf. [2–11]. There are several advantages for the potential method. For example, it can handle the possible discontinuity between different mediums very well and has good numerical accuracy; it avoids spurious solutions by adding a penalty function term in the dominant equation; it also has attractive features including natural coupling to moment and boundary element methods, global energy conservation. As far as we know, there are some relevant works on boundary data identification for the $\boldsymbol{H}$-equation, e.g.,[12–14]. In this paper, we are going to focus on determination of missing boundary data by means of the $\boldsymbol{A}$-ϕ method.

Assume that the boundary Γ is split into two complementary and non-empty parts $\Gamma_+ \subset \Gamma$ and $\Gamma_- = \Gamma \setminus \overline{\Gamma}_+$. Let Γ_- be unreachable and thus no information on the solution is available. In order to compensate for this lack of data we consider over-determined boundary data on Γ_+, namely,

$$\boldsymbol{E} \times \boldsymbol{n} = \widetilde{\boldsymbol{C}}_E \quad \text{and} \quad \boldsymbol{H} \times \boldsymbol{n} = \widetilde{\boldsymbol{D}}_H \quad \text{on } \Gamma_+. \tag{1.2}$$

The problem is to identify the appropriate boundary condition on Γ_- together with the magnetic and electric field in Ω. Now we decompose the electric field $\boldsymbol{E}$ of (1.1) into

$$\boldsymbol{E} = \boldsymbol{A} + \nabla\phi$$

to obtain the $\boldsymbol{A}$-ϕ equation

$$\varepsilon\partial_{tt}(\boldsymbol{A} + \nabla\phi) + \sigma\partial_t(\boldsymbol{A} + \nabla\phi) + \nabla \times \left(\frac{1}{\mu}\nabla \times \boldsymbol{A}\right) = -\partial_t \boldsymbol{J}_s \tag{1.3}$$

and afterwards, take divergence to the both hands of (1.3) to have

$$\nabla \cdot \Big(\varepsilon\partial_{tt}(\boldsymbol{A} + \nabla\phi) + \sigma\partial_t(\boldsymbol{A} + \nabla\phi)\Big) = -\nabla \cdot (\partial_t \boldsymbol{J}_s).$$

Then, by adding a penalty function term $-\nabla\big(\frac{1}{\mu}\nabla \cdot \boldsymbol{A}\big)$ into the dominant equation (1.3) to ensure divergence-free of the vector $\boldsymbol{A}$ (see, e.g. [15]), we obtain the following $\boldsymbol{A}$-ϕ formulation:

$$\begin{cases} \varepsilon\partial_{tt}(\boldsymbol{A} + \nabla\phi) + \sigma\partial_t(\boldsymbol{A} + \nabla\phi) + \nabla \times \left(\frac{1}{\mu}\nabla \times \boldsymbol{A}\right) - \nabla\left(\frac{1}{\mu}\nabla \cdot \boldsymbol{A}\right) = -\partial_t \boldsymbol{J}_s & \text{in } \Omega \times (0, T], \\ \nabla \cdot \Big(\varepsilon\partial_{tt}(\boldsymbol{A} + \nabla\phi) + \sigma\partial_t(\boldsymbol{A} + \nabla\phi)\Big) = -\nabla \cdot (\partial_t \boldsymbol{J}_s) & \text{in } \Omega \times (0, T]. \end{cases} \tag{1.4}$$

We replace the over-determined boundary data (1.2) by

$$\begin{cases} \boldsymbol{A} \times \boldsymbol{n} = \widetilde{\boldsymbol{C}}, \quad \frac{1}{\mu}\nabla \times \boldsymbol{A} \times \boldsymbol{n} = \widetilde{\boldsymbol{D}} & \text{on } \Gamma_+ \times (0, T), \\ \phi = 0 & \text{on } \Gamma \times (0, T), \end{cases} \tag{1.5}$$

and supplement the following boundary condition data:

$$\frac{1}{\mu}\nabla \cdot \boldsymbol{A} = 0 \quad \text{or} \quad \boldsymbol{A} \cdot \boldsymbol{n} = 0, \tag{1.6}$$

where,

$$\widetilde{\boldsymbol{C}} = \widetilde{\boldsymbol{C}}_E \quad \text{and} \quad \widetilde{\boldsymbol{D}} = -\partial_t \widetilde{\boldsymbol{D}}_H.$$

Let n be a positive integer and $\{t_i = i\tau : i = 0, \ldots, n\}$ be an equidistant partition of $[0, T]$ with $\tau = T/n$. Setting

$$u_i = u(\boldsymbol{x}_i), \qquad \delta u_i = \frac{u_i - u_{i-1}}{\tau}, \qquad \delta^2 u_i = \frac{\delta u_i - \delta u_{i-1}}{\tau},$$

the semi-discrete approximation to (1.4) reads: Given $(\boldsymbol{A}_0, \phi_0)$ and $(\delta\boldsymbol{A}_0, \delta\phi_0) = (\boldsymbol{A}'_0, \phi'_0)$, find $(\boldsymbol{A}_i, \phi_i)$ for $1 \le i \le n$ such that

$$\begin{cases} \varepsilon\delta^2(\boldsymbol{A}_i + \nabla\phi_i) + \sigma\delta(\boldsymbol{A}_i + \nabla\phi_i) + \nabla \times \left(\frac{1}{\mu}\nabla \times \boldsymbol{A}_i\right) - \nabla\left(\frac{1}{\mu}\nabla \cdot \boldsymbol{A}_i\right) = -\delta\boldsymbol{J}_{s,i}, \\ \nabla \cdot \Big(\varepsilon\delta^2(\boldsymbol{A}_i + \nabla\phi_i) + \sigma\delta(\boldsymbol{A}_i + \nabla\phi_i)\Big) = -\nabla \cdot (\delta\boldsymbol{J}_{s,i}). \end{cases} \tag{1.7}$$

Then we study the following static problem only instead of (1.7) in this paper.

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