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Periodic and solitary wave solutions for a generalized variable-coefficient Boiti–Leon–Pempinlli system

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ABSTRACT

In this paper, a generalized variable-coefficient Boiti–Leon–Pempinlli (BLP) system is studied via the modified Clarkson and Kruskal (CK) direct reduction method connected with homogeneous balance (HB) method, which can describe the water waves in fluid physics. A direct similarity reduction to nonlinear ordinary differential system is obtained. By solving the reduced ordinary differential system, new analytical solutions (including solitary and periodic types) in terms of Jacobi elliptic functions are given for the variable-coefficient BLP system.

1. Introduction

To describe the water waves in fluid physics, the $(2+1)$-dimensional Boiti–Leon–Pempinelli (BLP) system and BLP-type systems have been employed [1–3]. On the other hand, the variable-coefficient BLP equations can model many dynamic processes more realistically than their constant-coefficient counterparts. For example, one can consider taking into account the temporal evolution, nonuniform boundaries, and/or inhomogeneous media [4,5]. Corresponding to the $(2+1)$-dimensional BLP system, we will focus on the following generalized variable-coefficient BLP system [5,6]

$$\begin{aligned} u_{yt} &= \alpha(t)(u^2 - u_x)_{xy} + \beta(t) v_{xxx}, \\ v_t &= \gamma(t) v_{xx} + \lambda(t) u v_x, \end{aligned} \tag{1}$$

where $\alpha(t)$, $\beta(t)$, $\gamma(t)$, and $\lambda(t)$ are all the non-zero differentiable functions of t, $u(x, y, t)$ and $v(x, y, t)$ are, respectively, related to the horizontal velocity as well as the elevation of the water wave. On the $(2+1)$-dimensional variable-coefficient BLP system, water-wave symbolic computation has been performed in Ref. [5] and variable-coefficient-dependent auto-Bäklund transformation has been constructed, along with some variable-coefficient-dependent shock-wave-type solutions. Relevant variable-coefficient constraints have also been given with respect to water waves.

To handle the $(2+1)$-dimensional nonlinear evolution equations, such analytical methods as the Hirota bilinear method, binary Bell polynomial, symmetry methods, Bäklund transformation, Clarkson and Kruskal (CK) direct reduction method connected with homogeneous balance (HB) method have been used [7–24]. As a result, various analytical solutions and integrable properties can be studied. In this paper, a direct similarity reduction of system (1) to nonlinear ordinary differential system will be obtained. By solving the reduced ordinary differential system, new analytical solutions (including

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solitary and periodic types) in terms of Jacobi elliptic functions will be obtained for the variable-coefficient BLP system. Finally, discussions on the solutions and conclusions will be given.

2. The methodology

The modified enlarged CK direct reduction method connected with the HB method for the high dimensional nonlinear PDEs is given in the following details [20–24]:

Suppose we are given a nonlinear partial differential system for the two functions $u(t, x_i)$ and $v(t, x_i)$

$$P_j(x_i, t, u_t, u_{x_i}, u_{x_it}, v_t, v_{x_i}, v_{x_i,t} \ldots) = 0, \quad \text{where } (i = 1, 2, \ldots, r-1, \text{ and } j = 1, 2). \tag{2}$$

The modified enlarged method proceed in the following steps:

Step 1: We consider the solution of system (2) in the form

$$\begin{aligned} u &= \frac{\partial^n}{\partial x^n} U(w) + u_0, \\ v &= \frac{\partial^m}{\partial x^m} V(w) + v_0, \end{aligned} \tag{3}$$

where $w = w(t, x_i)$ are undetermined functions, u_0 and v_0 are two solutions of system (2).

According to the HB method n and m can be determined by balancing the linear term of the highest order derivative and the highest nonlinear terms of u and v in system (2).

Step 2: Substituting (3) into (2) and collecting all terms of U and V with same derivative and power. Step 3: To make the associated equations be nonlinear ordinary differential system in U and V, requiring ratios of their coefficients being functions of w, we obtain a set of undetermined equations for w, u_0, v_0 and other undetermined functions and by using the following two freedoms, without loss of generality we can determine those unknown functions with u_0, v_0 and w

(a) If u_0 has the form $u_0 = u_0(t, x_i) + \frac{\partial^n}{\partial x^n}\Omega$, then we can assume that $\Omega = 0$ (make the transformation $U(w) \to U(w) - \Omega$);
(b) If v_0 has the form $v_0 = v_0(t, x_i) + \frac{\partial^m}{\partial x^m}\Omega$, then we can assume that $\Omega = 0$ (make the transformation $V(w) \to V(w) - \Omega$);
(c) If $w(t, x_i)$ is defined by an equation of the form $\Omega(w) = w_0(x_i, t)$, we can also assume that $\Omega = w$ (make the transformation $w \to \Omega^{-1}(w)$).

3. Direct reduction for system (1)

In this section we have applied the modified enlarged CK method connected with the HB method on the variable-coefficients BLP system as follows:

Balancing the highest order derivative term and the nonlinear term of system (1), we obtain $n = m = 1$ so, we can suppose that the solution of system (1) takes the form

$$\begin{aligned} u(t, x, y) &= \frac{\partial}{\partial x} U + u_0, \\ v(t, x, y) &= \frac{\partial}{\partial x} V + v_0, \end{aligned} \tag{4}$$

where $U = U(w)$, $V = V(w)$, $w = w(t, x, y)$, $u_0 = u_0(t, x, y)$ and $v_0 = v_0(t, x, y)$ are functions to be determined later. Substituting (4) into (1), by collecting the same coefficients of U and V and its derivatives together and by using the coefficient of V'''' as the normalized coefficient then, the ratios of the coefficients of different derivatives and powers of U and V have to be functions of w say Γ_i $(i = 1, 2, \ldots, 18)$. As given here:

$$\begin{aligned} &\alpha w_x^3 w_y = \Gamma_1(w)\beta w_x^4, \\ &w_x w_t w_y - 2\alpha u_0 w_x^2 w_y + 3\alpha w_x^2 w_{xy} + 3\alpha w_x w_y w_{xx} = \Gamma_2(w)\beta w_x^4, \\ &w_{xy} w_t + w_x w_{ty} + w_y w_{xt} - 2\alpha u_{0y} w_x^2 - 2\alpha u_{0x} w_x w_y - 4\alpha u_0 w_x w_{xy} - 2\alpha w_y w_{xx} u_0 \\ &\quad + 3\alpha w_{xy} w_{xx} + 3\alpha w_x w_{xxy} + \alpha w_y w_{xxx} = \Gamma_3(w)\beta w_x^4, \\ &w_{xyt} - 2\alpha u_{0_y} w_{xx} - 2\alpha u_{0_x} w_{xy} - 2\alpha u_{0_{xy}} w_x - 2\alpha u_0 w_{xxy} + \alpha w_{xxxy} = \Gamma_4(w)\beta w_x^4, \\ &-2\alpha w_x^3 w_y = \Gamma_5(w)\beta w_x^4, \\ &-2\alpha(w_x^2 w_{xy} + w_x w_y w_{xx} + 2w_x^2 w_{xy} + w_x w_y w_{xx}) = \Gamma_6(w)\beta w_x^4, \\ &-2\alpha(w_{xx} w_{xy} + w_x w_{xxy}) = \Gamma_7(w)\beta w_x^4, \\ &-2\alpha w_x^3 w_y = \Gamma_8(w)\beta w_x^4, \qquad -6\beta w_x^2 w_{xx} = \Gamma_9(w)\beta w_x^4, \end{aligned}$$