Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Efficient and fast numerical method for pricing discrete double barrier option by projection method

Rahman Farnoosh [a,*], Amirhossein Sobhani [a], Mohammad Hossein Beheshti [b]

[a] School of Mathematics, Iran University of Science and Technology, 16844 Tehran, Iran
[b] Department of Mathematics, VaraminPishva Branch, Islamic Azad University, Iran

A R T I C L E I N F O

Article history:
Received 19 July 2016
Received in revised form 14 January 2017
Accepted 28 January 2017
Available online xxxx

Keywords:
Double barrier option
Projection methods
Black–Scholes model
Option pricing
Legendre polynomials

A B S T R A C T

In this paper, we introduce a new and considerably fast numerical method based on projection method in pricing discrete double barrier option. According to the Black–Scholes framework, the price of option in each monitoring dates is the solution of well-known partial differential equation that can be expressed recursively upon the heat equation solution. These recursive solutions are approximated by projection method and expressed in operational matrix form. The most important advantage of this method is that its computational time is nearly fixed against monitoring dates increase. Afterward, in implementing projection method we use Legendre polynomials as an orthogonal basis. Finally, the numerical results show the validity and efficiency of presented method in comparison with some others.

1. Introduction

Option pricing is one of the most interesting problems in mathematical finance. This field of study is being investigated from both theoretical and practical point of view by many researchers. Barrier options are one of the most applicable types of exotic derivatives in the financial market. As a description, a knock-out double barrier option is one that is deactivated when the price of underlying asset touches each of two predetermined barriers before the expiry date. According to the way of how the underlying asset price is monitoring, there are two types of barrier option, namely continuous and discrete. Discrete barrier options which we especially concern are those that the price of underlying asset is monitored at the specific dates, for example daily, weekly or monthly. Barrier options have been investigated by many researchers over the two past decades. Kamrad and Ritchken [1] applied the standard trinomial tree method and also Kwok [2] used the binomial and trinomial trees for pricing path-dependent options. Dai and Lyuu [3] introduced bino-trinomial tree method for pricing barrier options. In [4], the adaptive mesh model and its special case (AMM 8) were implemented and the quadrature methods QUAD K20 and QUAD K30 were proposed in [5]. An analytical solution for single barrier option based on Z-transform was driven by Fusai in [6]. A numerical solution for discrete barrier options based on combination of quadrature method and interpolation procedure is presented in [7]. Milev and Tagliani [8] presented a numerical algorithm for pricing discrete double barrier options. Golbabai et al. [9] applied finite element method for discrete double barrier option pricing. Farnoosh et al. [10,11] provided algorithms for pricing discrete single and double barrier options that are viable even for case of time-dependent parameters. Yoon and Kim [12] priced options under a stochastic interest rate by double Mellin transform. In [13], the

* Corresponding author.
E-mail addresses: rfarnoosh@iust.ac.ir (R. Farnoosh), a_sobhani@mathdep.iust.ac.ir, a_sobhani@aut.ac.ir (A. Sobhani), beheshti@iauvaramin.ac.ir (M.H. Beheshti).

http://dx.doi.org/10.1016/j.camwa.2017.01.019

Mellin transform is applied for pricing options with time-dependent parameters and discontinuous payoff. Fusai et al. [14] presented a method based on Wiener–Hopf factorization for pricing discrete exotic options.

The projection method is one of the most important applicable numerical methods that is implemented vastly for solving problems that arise in science, engineering, applied mathematics and especially in mathematical finance (see [15–17]). For example Galerkin, Petrov–Galerkin and collocation method are all projection methods that are widely used for solving integral equations, ordinary differential equations and partial differential equations. Projection operators, which have been investigated in functional analysis, are useful in discussing projection methods. In this article we present a numerical algorithm for pricing discrete double barrier options based on projection operators in general form.

This article is organized as follows. In Section 2, the Black–Scholes model and its corresponding partial differential equation for pricing discrete double barrier option are explained. Afterward by taking some well-known transformations, the problem is reduced to heat equation. Next the solution is driven as a recursive formula. A numerical approximation method based on projection operator is presented for pricing discrete double barrier option in Section 3. In Section 4, we use Legendre polynomials as an orthogonal basis in implementing projection method. Finally numerical results are given that demonstrate the proposed method is comparable with other numerical methods.

2. The pricing model

In this paper we assume that the stock price is accorded to prominent process, geometric Brownian motion, as follows:

$$dS_t = rS_t dt + \sigma S_t dB_t,$$

with initial stock price S_0, where coefficients r and σ are the risk-free rate and the volatility respectively. As mentioned in Section 1, we concern in pricing knock-out discrete double barrier call option on stock, i.e. a call option that becomes worthless if the stock price hits lower or upper barrier at the specific monitoring dates $0 = t_0 < t_1 < \cdots < t_M = T$. According to the well-known Black–Scholes framework, the price of discretely monitored double barrier call option as a function of stock price at time $t \in (t_{m-1}, t_m)$, namely $\mathcal{P}(S, t, m-1)$, is obtained from forward solving the following partial differential equations [18]

$$-\frac{\partial \mathcal{P}}{\partial t} + rS\frac{\partial \mathcal{P}}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \mathcal{P}}{\partial S^2} - r\mathcal{P} = 0, \tag{1}$$

with the following initial conditions:

$$\mathcal{P}(S, t_0, 0) = (S - E)\,\mathbf{1}_{(\max(E,L) \le S \le U)},$$
$$\mathcal{P}(S, t_m, 0) = \mathcal{P}(S, t_m, m-1)\,\mathbf{1}_{(L \le S \le U)}; \quad m = 1, 2, \ldots, M-1,$$

where $\mathcal{P}(S, t_m, m-1) := \lim_{t \to t_m} \mathcal{P}(S, t, m-1)$. Also the constants E, L and U are exercise price, lower barrier and upper barrier respectively. Following simple change of variable is performed in two steps. At first we define function $C(z, t, m)$ as below:

$$C(z, t, m) := \mathcal{P}(S, t, m),$$

where $z = \ln\left(\frac{S}{L}\right)$; $E^* = \ln\left(\frac{E}{L}\right)$; $\mu = r - \frac{\sigma^2}{2}$; $\theta = \ln\left(\frac{U}{L}\right)$ and $\delta = \max\{E^*, 0\}$. Then the partial differential equation (1) and its initial condition are changed into:

$$-C_t + \mu C_z + \frac{\sigma^2}{2} C_{zz} = rC, \tag{2}$$
$$C(z, t_0, 0) = L\left(e^z - e^{E^*}\right)\mathbf{1}_{(\delta \le z \le \theta)},$$
$$C(z, t_m, m) = C(z, t_m, m-1)\,\mathbf{1}_{(0 \le z \le \theta)}; \quad m = 1, 2, \ldots, M-1.$$

As a second step we use the following transformation:

$$C(z, t_m, m) = e^{\alpha z + \beta t} h(z, t, m),$$

where

$$\alpha = -\frac{\mu}{\sigma^2}; \qquad c^2 = -\frac{\sigma^2}{2}; \qquad \beta = \alpha\mu + \alpha^2\frac{\sigma^2}{2} - r.$$

Therefore, the partial differential equation (2) and its initial condition are led to:

$$-h_t + c^2 h_{zz} = 0,$$
$$h(z, t_0, 0) = Le^{-\alpha z}\left(e^z - e^{E^*}\right)\mathbf{1}_{(\delta \le z \le \theta)}; \quad m = 0,$$
$$h(z, t_m, m) = h(z, t_m, m-1)\,\mathbf{1}_{(0 \le \theta \le z)}; \quad m = 1, \ldots, M-1.$$

Please cite this article in press as: R. Farnoosh, et al., Efficient and fast numerical method for pricing discrete double barrier option by projection method, Computers and Mathematics with Applications (2017), http://dx.doi.org/10.1016/j.camwa.2017.01.019