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# Existence of multiple solutions for modified Schrödinger–Kirchhoff–Poisson type systems via perturbation method with sign-changing potential

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ABSTRACT

In this paper, we prove the existence of positive solutions and negative solutions for the following modified Schrödinger–Kirchhoff–Poisson type systems

$$\begin{cases} -\left(a+b\int_{\mathbb{R}^3} |\nabla u|^2\right)\Delta u + V(x)u + \phi u - \frac{1}{2}u\Delta(u^2) = f(x,u), & \text{in } \mathbb{R}^3, \\ -\Delta\phi = u^2, & \text{in } \mathbb{R}^3, \end{cases}$$

where $a > 0$, $b \geq 0$, $V$ and $f$ are continuous functions and $V(x)$ is allowed to be sign-changing. Under some certain assumptions on $V$ and $f$, we prove the existence of nontrivial nonnegative solutions, nontrivial nonpositive solutions and sequence of high energy solutions via the perturbation method.



## 1. Introduction and preliminaries

This article is concerned with a class of modified Schrödinger–Kirchhoff–Poisson system

$$\begin{cases} -\left(a+b\int_{\mathbb{R}^3} |\nabla u|^2\right)\Delta u + V(x)u + \phi u - \frac{1}{2}u\Delta(u^2) = f(x,u), & \text{in } \mathbb{R}^3, \\ -\Delta\phi = u^2, & \text{in } \mathbb{R}^3, \end{cases} \tag{1.1}$$

where $a > 0$, $b \geq 0$, $V$ and $f$ are continuous functions and $V(x)$ is allowed to be sign-changing.

Very recently, Batkam and Júnior [1] studied the following modified Schrödinger–Kirchhoff–Poisson system in $\Omega$,

$$\begin{cases} -\left(a+b\int_{\mathbb{R}^3} |\nabla u|^2\right)\Delta u + \phi u = f(x,u), & \text{in } \Omega, \\ -\Delta\phi = u^2, & \text{in } \Omega, \\ u = \phi = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

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where $\Omega$ is a bounded smooth domain of $\mathbb{R}^N$ with $N = 1, 2$ or 3 and $f : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ is a continuous function. The authors [1] proved that the problem (1.2) has at least three solutions: one positive, one negative, and one which changes its sign. Furthermore, in case $f$ is odd with respect to $u$, the authors obtained unbounded sequence of sign-changing solutions. Moreover, if $a = 1$ and $b = 0$ in (1.2), Ruiz and Siciliano [2] proved the existence of multiplicity of solutions depending on two parameters and $f(x, u) = |u|^{s-1}s$, where $s \in (1, 5)$. For more references of system (1.2), please see [3–6] and references therein.

If we choose $a = 1$ and $b = 0$, then (1.1) reduces to the following modified Schrödinger–Poisson system

$$\begin{cases} -\Delta u + V(x)u + \phi u - \frac{1}{2}u\Delta(u^2) = f(x, u), & \text{in } \mathbb{R}^3, \\ -\Delta\phi = u^2, & \text{in } \mathbb{R}^3. \end{cases} \tag{1.3}$$

It is well known that (1.3) has been already studied via a perturbation method and Feng and Zhang [7] proved the existence of positive solutions, negative solutions by applying Mountain Mass theorem.

In the recent years, there are many papers dealing with the following modified Kirchhoff-type equation

$$-\left(a + b\int_{\mathbb{R}^3} |\nabla u|^2\right)\Delta u + V(x)u - u\Delta(u^2) = f(x, u), \quad \text{in } \mathbb{R}^3. \tag{1.4}$$

In particular, Feng, Wu and Li [8] proved some existence results for positive solutions, negative solutions and sequence of high energy solutions via perturbation method with 4-superlinear nonlinearity on $f$. Afterwards, Wu et al. [9] studied the existence of infinitely many small energy solutions with local nonlinearities by applying Clask's Theorem to a perturbation functional. If we delete the quasilinear term $u\Delta(u^2)$, then there are many papers to study this problem. For this problem, we refer the reader to [10,11] and the references therein.

As far as we know, the paper [12] firstly used perturbation method to study the existence of solutions for quasilinear Schödinger equation in bounded smooth domain. Following the work of [12], Liu, Liu and Wang [13] proved the existence of multiple sign-changing solutions for generalized quasilinear Schödinger equation in bounded smooth domain. Subsequently, there are also many classical and interesting studies on quasilinear Schödinger equation and generalized quasilinear Schödinger equation in $\mathbb{R}^N$, see for instance [14–24] and the references therein.

With regard to sign-changing potentials on $V$, there are many papers studying this problem for different equations, see e.g. [25–29,10,30–34]. We consider the existence of multiple solutions for the problem (1.1) with sign-changing potentials. As far as we know, there are rarely papers to study the existence of multiple solutions for the problem (1.1) with sign-changing potential.

Next, we will use the following notation. Let

$$H^1(\mathbb{R}^3) = \left\{u \in L^2(\mathbb{R}^3) : \nabla u \in L^2(\mathbb{R}^3)\right\}$$

endowed with the norm

$$\|u\|_{H^1} = \left(\int_{\mathbb{R}^3} (|\nabla u|^2 + u^2)dx\right)^{1/2}$$

and the inner product

$$(u, v)_{H^1} = \int_{\mathbb{R}^3} (\nabla u \nabla v + uv)dx.$$

Set

$$H^1_V(\mathbb{R}^3) = \left\{u \in H^1(\mathbb{R}^3) : \int_{\mathbb{R}^3} \widetilde{V}(x)u^2 dx < +\infty\right\}$$

endowed with the norm

$$\|u\|_{H^1_V} = \left(\int_{\mathbb{R}^3} (|\nabla u|^2 + \widetilde{V}(x)u^2)dx\right)^{1/2}$$

and the inner product

$$(u, v)_{H^1_V} = \int_{\mathbb{R}^3} (\nabla u \nabla v + \widetilde{V}(x)uv)dx$$

and $W^{1,4}(\mathbb{R}^3)$ endowed with the norm

$$\|u\|_{W^{1,4}} = \left(\int_{\mathbb{R}^3} (|\nabla u|^4 + u^4)dx\right)^{1/4}.$$

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