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The singular boundary method for two-dimensional static thermoelasticity analysis

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ABSTRACT

Based on the singular boundary method (SBM) coupled with the radial basis functions (RBF), a meshless algorithm is proposed to solve two-dimensional static thermoelasticity problems. The displacements and stresses can be written as the sum of a homogeneous solution and a particular solution. The RBF and SBM are used to construct the corresponding approximate particular and homogeneous parts, respectively. All unknowns are determined by satisfying the governing equations and the boundary conditions. Numerical experiments are performed for different cases, and the proposed meshless method is effectively validated by comparing the results with the available analytical solutions.

1. Introduction

Thermoelastic stress analysis describes the relation between the stress of an elastic body and its temperature variations. Numerous engineering problems in practice are subjected to thermal loadings. The current static thermoelastic analysis is a starting point for the various generalizations including visco-thermoelasticity, thermoelasticity with thermal diffusion, and thermoelasticity with finite wave speeds. Before extending the current methodology to these theories, it is beneficial to research the simpler case of static thermoelasticity in depth.

The basic theory for thermoelasticity and its analytical solution can be referred to some classic books [1–3]. Although analytical approaches can provide closed-form solutions, they are always limited to simple geometries, specific types of boundary conditions, and certain loading cases. To perform a general analysis, numerical simulation is necessary.

Mathematical problems of thermoelasticity have been solved by many numerical techniques such as the finite element method (FEM) [4–6], the boundary element method (BEM) [7–11], the dual reciprocity BEM (DRBEM) [12], the method of fundamental solutions (MFS) coupled with radial basis functions (RBF) [13–16], the moving least-squares method combined with the local boundary integral method [17], and the Galerkin BEM combined with RBFs [18]. Among the above-mentioned methods, the BEM is a promising numerical method due to its high accuracy, applicability to infinite domains, and ability to capture singularities with less discretization effort. However, for thermoelastic analysis, the conventional BEM involves a domain integral which is incongruent with its main attraction of boundary-only discretization. To deal with this shortcoming, the DRBEM and the particular integral method [19], which is equivalent to DRBEM, have been developed.

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Serving as an alternative to the BEM, the meshless singular boundary method (SBM) [20] has recently been developed. Its advantages are that is free of mesh and integrations, and its effective treatment of complicated load conditions. The origin intensity factors (OIF) are introduced in the SBM to isolate the singularity of the fundamental solutions thus avoiding the fictitious boundary in the MFS. The SBM has been successfully applied for solving heat conduction [21–23], linear elasticity [24,25], viscoelastic wave [26], and dynamic poroelastic [27] problems.

The objective of the paper is to further develop a meshless algorithm, based on the SBM and RBF approximation, for analyzing two-dimensional thermoelasticity problems. According to the method of particular solutions (MPS) [28,29] and the dual reciprocity method (DRM) [30,31], the solutions for displacements and stresses can be written as the sum of a homogeneous solution and a particular solution. In this paper, we adopt a new special RBF to simulate the particular solution and apply the SBM to obtain the homogeneous solution. It should be noted that the singularity in the special RBF is settled firstly by using the concept of the OIF.

The rest of this paper is organized as follows. Section 2 introduces the mathematical formulation of the two-dimensional thermoelasticity problem in an isotropic medium. The proposed scheme and its numerical implementation are reviewed in Section 3. In Section 4, the performance of the proposed scheme is investigated on three benchmark examples. Finally, some conclusions and remarks are provided in Section 5.

2. Mathematical formulation

In this section, the basic equations for general two-dimensional thermoelasticity problems are briefly reviewed.

Consider a linear–elastic homogeneous, mechanically and thermally isotropic solid occupying a simply connected domain Ω bounded by a smooth boundary Γ. The solid is characterized by the following material constants: the thermal conductivity, κ, coefficient of linear thermal expansion, α, Poisson's ratio, ν, and Young's modulus, E. For two-dimensional isotropic thermoelasticity without a gravity body force, the governing equations are given by

$$\sigma_{ij,j} = \sum_{j=1}^{2} \frac{\partial^2 \sigma_{ij}}{\partial x_j} = 0, \tag{1}$$

$$\sigma_{ij} = \bar{\lambda}\delta_{ij}\varepsilon_{kk} + 2\bar{\mu}\varepsilon_{ij} - \bar{\gamma}\delta_{ij}T, \tag{2}$$

where $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ are the Cauchy strains, $u_{i,j} = \frac{\partial u_i}{\partial x_j}$, $\bar{\lambda} = \frac{2\bar{\nu}}{1-2\bar{\nu}}\bar{\mu}$ denotes the lame constant, $\bar{\mu} = \frac{\bar{E}}{2(1+2\bar{\nu})}$ is the shear modulus, $\bar{\gamma} = \frac{\bar{\alpha}\bar{E}}{1-2\bar{\nu}}$ is the thermoelastic coefficient, $\bar{E}$, $\bar{\nu}$ and $\bar{\alpha}$ have different values for plane stress and strain states, respectively, such as,

$$\begin{cases} \bar{E} = E \qquad \bar{\nu} = \nu \qquad \bar{\alpha} = \alpha \quad \text{for plane strain,} \\ \bar{E} = \dfrac{1+2\nu}{(1+\nu)^2}E \qquad \bar{\nu} = \dfrac{\nu}{1+\nu} \qquad \bar{\alpha} = \dfrac{1+\nu}{1+2\nu}\alpha \quad \text{for plane stress.} \end{cases} \tag{3}$$

It should be noted that the index j in Eq. (1) or the index k in Eq. (2) on which a summation is carried out is called a dummy index. Similarly, the indexes l, j and k in the following Eqs. (6)–(11) denote dummy indexes as well. A dummy index may be replaced by any other dummy index, for example, $A_{ii} = A_{jj} = A_{11} + A_{22}$.

In the absence of heat sources, the equation governing steady-state heat conduction satisfies

$$-\nabla \cdot (\kappa \nabla T) = 0, \quad \text{in } \Omega. \tag{4}$$

The thermo-elasticity problems governed by Eqs. (1), (2) and (4) have to be solved subject to the following boundary conditions

$$T = \tilde{T}, \quad \text{on } \Gamma_T, \tag{5a}$$

$$q = -(\kappa \nabla T) \cdot \mathbf{n} = \tilde{q}, \quad \text{on } \Gamma_q, \tag{5b}$$

$$\mathbf{u} = \tilde{\mathbf{u}}, \quad \text{on } \Gamma_u, \tag{5c}$$

$$\mathbf{t} = \boldsymbol{\sigma}\mathbf{n} = \tilde{\mathbf{t}}, \quad \text{on } \Gamma_t, \tag{5d}$$

where $\tilde{T}$, $\tilde{q}$, $\tilde{\mathbf{u}}$, and $\tilde{\mathbf{t}}$ are the prescribed temperatures, heat fluxes, displacements and tractions, respectively. The vector $\mathbf{n}$ represents the outward unit normal vector to the physical boundary Γ. The governing Eqs. (1)–(2) in terms of displacement and temperature can be expressed as

$$\left(\bar{\lambda} + \bar{\mu}\right) u_{j,ji} + \bar{\mu}u_{i,jj} - \bar{\gamma}T_{,i} = 0, \quad \text{in } \Omega, \tag{6}$$

where $u_{j,ji} = \sum_{j=1}^{2} \frac{\partial^2 u_j}{\partial x_j \partial x_i}$, $u_{i,jj} = \sum_{j=1}^{2} \frac{\partial^2 u_i}{\partial x_j^2}$, $i = 1, 2$.