Computers and Mathematics with Applications 72 (2016) 2751–2772

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Alternative kinetic theory based lattice Boltzmann model for incompressible axisymmetric flows

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ARTICLE INFO

Article history:
Received 6 March 2016
Received in revised form 11 July 2016
Accepted 30 September 2016
Available online 24 October 2016

Keywords:
Axisymmetric Boltzmann equation
Kinetic theory based model
Incompressible flows

ABSTRACT

Based on the axisymmetric Boltzmann equation, an incompressible lattice Boltzmann model for axisymmetric flows is proposed within the framework of the kinetic theory based model developed by Guo et al. (2009). While retaining the advantages of the Guo et al. (2009) model in terms of the solid physics basis and simple source terms involving no gradient calculations, the present model further improves the numerical stability, and reduces compressibility errors and computational requirements. Armed with the assumption that the fluid density is a constant and thus the fluid pressure has no direct relation with the density, the incompressibility conditions are realized by applying the Hermite expansion. Then, the present model employs a novel way of calculating the fluid pressure which is derived from the modified second-order moment equation. Additionally, based on the regularized lattice BGK (RLBGK) model, an extra relaxation parameter pertaining to the ghost mode is introduced to enhance the numerical stability of the present model. The accuracy and applicability of the present model are verified by both the Chapman–Enskog theoretical analysis and numerical validations. It is demonstrated via well-acknowledged test cases that the present model is accurate and reliable for incompressible axisymmetric flows, and is able to effectively reduce the compressibility errors vis-à-vis the Guo et al. (2009).

1. Introduction

The lattice Boltzmann (LB) equation, originating from the lattice gas micro-dynamics, provides us with a convenient alternative for the conventional computational method based on the direct discretization of the Navier–Stokes (N–S) equations [1–5]. The resulting LB method has advantageous features such as a simple evolution procedure, intrinsic parallel nature and the easy treatment of boundary conditions for systems with complex geometry. Besides, the LB equation can also be regarded as the discretization of the Boltzmann equation [6]. From this perspective, the LB method is claimed to be a second-order explicit finite difference approximation of the relevant macroscopic equations, with the adopted finite difference stencils determined by the relevant lattice models [7,8]. Moreover, despite the relevance between the LB method and the macroscopic N–S equations, the applications of the LB method have been successfully extended to some more

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http://dx.doi.org/10.1016/j.camwa.2016.09.028
0898-1221/

complicated systems beyond the N–S solutions, such as the interfacial dynamics [9,10], flows at small scales [11–13] and non-equilibrium flows [14,15].

Axisymmetric fluid flows, and heat and mass transfer in cylindrical systems are widely encountered in engineering practices and can be directly resolved with the three-dimensional (3D) LB method [16–18]. However, in such applications, the axisymmetric conditions are neglected, otherwise the 3D axisymmetric problems can be reduced to quasi-2D ones in the meridian plane and the computation efficiency is highly enhanced. Two strategies are available for designing the axisymmetric LB models [19]. First, at the macroscopic level, the axisymmetric N–S equations in cylindrical systems are represented in a pseudo-Cartesian system with the extra terms regarded as source terms which are then incorporated into the standard LB equation. Second, the other strategy is realized at the distribution function level, including the modifications of the LB equation and the relevant equilibrium distribution functions. Harnessing the first strategy, the first axisymmetric LB model was proposed by Halliday et al. [20]. It was reported that the recovered momentum equations were incorrect with a missing term related to the radial velocity [21–23]. Such mistakes were removed by the following models proposed by Lee et al. [21,24] and Reis and Phillips [22,23], which were able to predict the accurate radial velocity distributions. The model proposed by Halliday et al. [20] achieved widespread applications, for example, in axisymmetric multiphase flows [25–27] and rotating thermal flows, whereby a hybrid method involving the second-order finite difference method was used for describing the azimuthal velocity and temperature evolution [28]. However, as stated by Zhou [29], the source terms in these earlier models were rather complex; for example, in the method of Halliday et al. [20], as many as five terms were involved in the second forcing term (which is related to the axisymmetric momentum equation), and the resulting forcing term expression contains more than ten items when the missing term (which improves the accuracy of the model) was added. In order to simplify the source terms, Chen et al. [30,31] derived an axisymmetric LB model from the vorticity stream equation, but the drawbacks of the Chen et al. [30,31] model were that the vorticity at the boundaries was hard to determine, and that a Poisson equation had to be solved at each time step, which significantly decreased the computation efficiency. As another means to simplify the source terms, Zhou [29] developed a LB model based on the standard LB equation, in which the source terms were simply the extra term in the transformed pseudo-Cartesian macroscopic equations and a centered scheme was adopted to remove the discrete lattice effect of the forcing terms.

Despite the efforts in simplifying the source terms, the velocity gradient terms in the source terms cannot be removed by above models. In the axisymmetric thermal LB model proposed by Li et al. [32], an additional collision term, in which the relaxation parameter was correlated to the discrete particle velocity vector, was introduced and the temperature gradient terms were avoided in the source terms. Subsequently, an improved axisymmetric LB model was developed by the same authors [33], who showed that the gradient calculation in the source terms is realized by the non-equilibrium part of the distribution. This idea [33] was later introduced into the Zhou [34] model, in which velocity gradients in the force terms were eliminated.

It should be noted that above models have all developed based on the idea from the first strategy stated above, in which the standard LB equation for the N–S equations in Cartesian systems was applied. Regarding the second strategy implemented at the distribution function level, an axisymmetric LB model was proposed by Guo et al. [35] from the axisymmetric continuous Boltzmann equation. Guo et al. [35] described the radial, axial and azimuthal velocity components in the same fashion, and the source terms naturally contained no velocity gradients. Notably, the kinetic theory origin implies a solid physics basis, which is a clear advantage. Subsequently, the numerical stability of the Guo et al. [35] model was enhanced by replacing the BGK collision operator with the multi-relaxation-time operator [36], after which the application was extended to axisymmetric thermal flows [37,38] and multiphase flows [39].

It has been demonstrated via the Chapman–Enskog analysis that the derived equations from the Guo et al. model can be transformed to the standard axisymmetric N–S equations by applying the incompressible continuity equation [35]. However, two shortcomings in the Guo et al. [35] model lies in: (i) the ideal gas state equation adopted leads to the compressible continuity equation, which in turn leads to compressibility errors, and (ii) the source terms were not expressed in a determinate way, and only one set of possible expressions was presented. To address the shortcomings, in this study, the moment equations of the distribution functions are firstly modified in accordance with the incompressibility conditions, i.e., the fluid density is assumed to be a constant such that the fluid pressure evolves independently. The Chapman–Enskog analysis in the Appendix demonstrates that the standard axisymmetric N–S equations for incompressible flows are obtained from the axisymmetric Boltzmann equation with the modified moments. Then, the expressions of the equilibrium distribution functions are derived from the relevant modified moments by applying the moment method developed by Grad [40,41]. Similarly, the source terms of the present model are determined with the moment constraints derived from the Chapman–Enskog analysis. Thereafter, the discretization in phase space for the present model is implemented via different means. Whereas the evolution equations for the two reduced distribution functions are both discretized by the D2Q9 lattice in the Guo et al. [35] model, the present model is such that the distribution function for the axial and radial velocity components evolves on the D2Q9 lattice, while the distribution function for azimuthal velocity is discretized by the D2Q4 lattice model. The overall accuracy of the present model is not only unaffected by the simplified phase space discretization, but the computation efficiency is also highly enhanced. Lastly, a new set of computing formulas for the independent macroscopic variables, i.e., the fluid velocity and pressure, emanates from the modified moment equations in the present model.

Furthermore, motivated by the concept of the regularized lattice BGK (RLBGK) model [42,43], the evolution of the ghost modes in the present model is suppressed by adding an extra relaxation parameter pertaining to the non-hydrodynamic