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Three-dimensional stress analysis of thin structures using a boundary element method with sinh transformation for nearly singular integrals

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ABSTRACT

In this work a three dimensional (3D) boundary element method was established with an efficient nonlinear coordinate transformation scheme, namely sinh transformation, to evaluate nearly singular integrals in boundary integral formulations. Second-order quadrilateral surface elements were developed based on this method to accurately describe the geometry of thin structures. The elastic behaviors of selected thin structures were then computed by using the 3D boundary element model to demonstrate the accuracy and efficiency of this approach. A number of testing examples, i.e., the 3D Kirsch problem, the thin spherical shell problem, the ellipsoidal vessel problem with non-uniform thickness and the hollow circular cylinder problem, were numerically studied to test the established method. Remarkable accuracy and efficiency were found in the developed approach through the comparison to the numerical results and analytical solutions reported in the literature.

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1. Introduction

The use of thin structures attracted considerable attention over the past decades. The breakthroughs in micro/nano-fabrication technology have allowed a great number of ultra-thin structures to be designed and utilized in microelectronic and coating industries [1–6]. However, the stress analysis of thin structures is quite challenging. For instance, the drastic difference in the length scales of ultra-thin films (with the thickness-to-length ratio under 10^{-3}) may raise problematic issues to conventional finite element method (FEM) during the analysis [7–9]. The reason lies in the fact that the accuracy and reliability of the FEM results cannot be guaranteed for such ultra-thin structures in the micro- or nano-scales [1,10,11]. In addition, to maintain the acceptable element aspect ratio, a large number of elements must be used in the FEM analysis. The computation procedure thus requires too much preprocessing and CPU time as the thickness gets very low [11].

The boundary element method (BEM), as an alternative approach, was rapidly developed during the past two decades. It is considered to be a competitive method in solving thin-structure problems [12–14]. However, challenges still exist in the application of BEM. One of the major difficulties is the numerical evaluation of nearly singular integrals during the analysis of thin structures [10,15]. The integrals are termed as nearly singular integrals when the collocation point is very close to, but not on, the element. Although these integrals are actually regular in theory, they cannot be accurately evaluated by using standard Gaussian quadrature rule [16,17]. Effective and accurate evaluation of nearly singular integrals is thus a key to the application of BEM for thin structures. A number of numerical approaches have been developed

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to eliminate near-singularities. Most of the developed work is focused on the studies of nonlinear transformation. The approaches developed so far include, but are not limited to, cubic polynomial transformation [18], degenerate mapping method [19], coordinate optimal transformation [20], sigmoidal transformation [21,22], sinh transformation [11,23–27], rational transformation [28], distance transformation [29], exponential transformation [16,17] and nonlinear coordinate transformation [1,30]. More detailed surveys of the earlier work are given by Zhang et al. [17].

Among the existing studies, the sinh-transformation method proposed by Johnston and Elliott [23] has been demonstrated to be of high accuracy for the numerical evaluation of nearly singular integrals. One of the major advantages of this method is that it can automatically take into account the position of the nearly singular point and the distance from the source point to the element. In addition, the sinh-transformation method is applicable to a broad range of integrals without extra computational effort. It is shown that this approach is very accurate [23,24,26] and, in most cases, superior to most of existing methods in terms of overall accuracy and stability [25]. In a more recent work [27], an improved sinh-transformation method was established based on the second-order quadrilateral surface element. The major advantage of this developed method lies in that it employs an accurate distance formula in the exponential transformation method [17], which is used in sinh transformation. Such distance formula is able to accurately calculate the actual distance from the evaluation point to a generic point of the second-order quadrilateral surface element, which will otherwise have to be evaluated by first-order Taylor expansion approximation. It is anticipated that the nearly-singular-integral problem in 3D BEM can be solved more efficiently and accurately by employing the improved sinh-transformation method.

The accuracy and applicability of the improved sinh-transformation approach established in [27] were preliminarily evaluated based on the BEM analysis of 3D boundary-layer potential problems. We feel that the investigation on this mathematical approach may be concluded only after its performance (e.g., accuracy, efficiency and applicability) is further evaluated on certain types of key problems. Comparison of the results based on the current approach and existing ones has to be made on carefully designed benchmark problems. The thin-structure elasticity problem is one of the most important and major representative problems in BEM analysis. According to our best knowledge, however, no related study was reported regarding the implementation of such approach in solving 3D thin-structure elasticity problems or any other problems arising from engineering practice. To explore such field and to make the investigation on this mathematical approach more complete, we feel that it is urgently needed to extend this sinh-transformation approach to the evaluation of nearly singular integrals appearing in boundary integral equations of 3D elasticity problems, in particular for thin structures.

In this work, a general boundary element framework is established with the improved sinh-transformation approach for nearly singular integrals [27] to study the mechanical behavior of four selected representative elasticity problems, i.e., the 3D Kirsch problem, the thin spherical shell problem, the ellipsoidal vessel problem with non-uniform thickness and the hollow circular cylinder problem. It should be noted that the problems being investigated in the present work are generally considered more challenging in BEM computation. (i) The order of near singularities appearing in the discretized form of boundary integral equations of 3D elasticity is higher than that of 3D potential problem. (ii) More crucially, for thin-structure problems, the corresponding meshes generated on the two opposite boundaries of a thin structure are extremely close to each other. Thus, the singular and nearly singular integrals on the boundary need to be evaluated simultaneously when solving the unknown boundary variables. This is a scenario that was not present in earlier computation with the sinh-transformation approach [27]. (iii) In thin-structure problems, the need of calculation of nearly singular integral is inevitable for nearly all the interior points since these points are all very close to the boundaries. This essentially added complexity to the present investigation. To tackle these challenges, the second-order quadrilateral surface element was employed to adequately characterize the boundary geometry of the thin structures. The improved sinh-transformation method will be extended and utilized to numerically evaluate the nearly singular integrals in the boundary integral equations. The accuracy and efficiency of the presented approach are being evaluated.

This paper is organized as follows: following this introduction, the basics of the sinh-transformation approach are introduced in Section 2; in Section 3, the developed approach is applied to solve the selected 3D elasticity problems and the accuracy and efficiency of the approach are examined and discussed; and the conclusions and remarks are finally provided in Section 4.

2. The basics of this sinh-transformation approach

2.1. A mathematical description of the nearly singular integral

In this section, the general form of a nearly singular integral in the boundary integral formulation is to be described. The boundary integral equation of a 3D linear-elasticity problem is first considered. In the absence of body forces, the well-known self-regular boundary integral equation (BIE) for 3D elasticity problem can be written as [16]:

$$0 = \int_{\Gamma} \left[\left(u_j(\boldsymbol{x}) - u_j(\boldsymbol{y}) \right) T_{ij}(\boldsymbol{x}, \boldsymbol{y}) \right] d\Gamma - \int_{\Gamma} t_j(\boldsymbol{x}) U_{ij}(\boldsymbol{x}, \boldsymbol{y}) d\Gamma. \tag{1}$$

In Eq. (1), u_j and t_j are the Cartesian components of the displacement vector and traction vector, respectively. U_{ij} and T_{ij} are Kelvin displacement and traction fundamental solutions, respectively. The vector $\boldsymbol{x}$ (or $\boldsymbol{y}$) represents the position of

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